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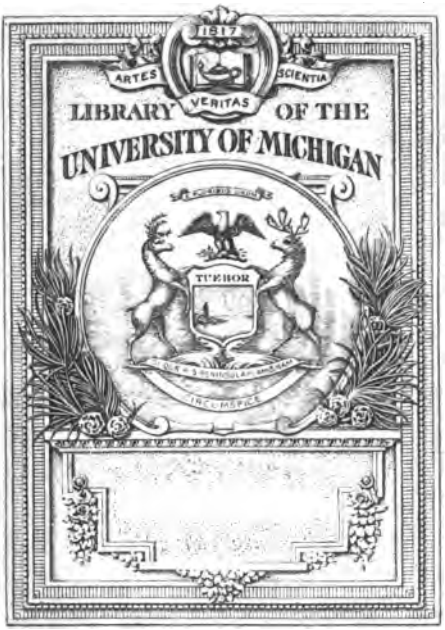
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AN EASY

INTRODUCTION

TO PLANE

TRIGONOMETRY:

THE

Application of it to the measuring of

HEIGHTS, DISTANCES and LAND;

AND THE

Use of the portable CASE of

MATHEMATICAL INSTRUMENTS.

By C. ASHWORTH.

SALOP:

Printed by J. EDDOWES: And sold by J. BUCKLAND, in *Pater-noster-row*, LONDON.

MDCCLXVI.

Advertiſement.

AS the articles which are the ſubject of the following ſheets have been handled by many conſiderable writers, it ſcarce can be imagined that any thing new can be added: at leaſt the author of this eſſay makes no ſuch pretence. His only aim was to draw together what he found on the ſubject (generally in large volumes) into a ſmall compaſs, and place it in an eaſy light. It was firſt deſigned only for his own information, for which reaſon he tranſcribed the ſentiments, and frequently the very words, of his authors without heſitation. When he ſent it to the preſs, he only intended to have a few copies printed for the young gentlemen under his care; and, tho' he has ſince permitted a few to be expoſed to ſale, he has uſed no art to conceal the plagiariſm; with which he hopes no one will be ungenerous enough to upbraid him, after he has ſo frankly confeſſed it.

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Explanation of the Algebraic marks.

$+$ more, or addition; as $1 + 2$, 1 more 2, or 1 and 2 added, or taken together.

$-$ leſs, or ſubſtraction; as $6 - 4$, 6 leſs 4, or what remains of 6 when 4 is ſubſtracted from it.

$\times$ into, or multiplication; as 6×4, 6 into 4, or the product of 6 multiplied by 4.

$\div$ by, or diviſion; as $6 \div 2$, 6 by 2, or the quotient ariſing from dividing 6 by 2.

$:: ::$ proportion; as $A : B :: C : D$, A is to B as C is to D.

$=$ equal; as $A = B$, A is equal to B.

^{2} the ſquare; as 6^2 the ſquare of 6, or 6×6.

$^\circ$ a degree; as 6° is 6 degrees.

$'$ a minute, as $4'$ is 4 minutes.

ERRATA.

Page 4. l. 14. after 90° insert a colon.
P. 6. l. 21. for A B O, *read* A B.
P. 7. l. 3. for secant arcs, *read* seants of the arcs.
P. 8. l. 20. for 209, *read* 2 O G.c
ib. l. 24. for cons, *read* co-s.
P. 9. l. 26. for l.)47, *read* l. 47).
P. 10. l. 23. for B A, *read* B D.
P. 13. l. 14. for *angles*, F E H, *read angles*; F E A.
P. 14. l. 22. for As to the co-fine, co-tangents, *read* As the co-fine, co-tangent.
P. 25. l. penult. for tens, *read* tenths.
P. 27. l. 13. *dele*) line 15. *dele*) l. 21. *dele* ,
P. 29. l. 5. for A C or B D, *read* E A or E B.
P. 37. l. 11. for 60°, *read* 65°.
P. 42. l. 25. for (14-3), *read* (143).
P. 43. l. 14. for only *so*, *read* only half so. ib. l. 21. for 51°, *read* 31°.
P. 45. l. 3. for segments, *read* segment.
ib. l. 19 and 20. *read* to B D — D C (difference of seg.) 49 1.68948 2) B C = 87.

AN EASY

INTRODUCTION

TO PLANE

TRIGONOMETRY.

PART I.

Definitions and obſervations.

DEF. I.

PLANE TRIGONOMETRY teaches the method of meaſuring the ſides and angles of a plane triangle, when ſome of them are already known.

SCHOL.

The ſides of a triangle are denoted by meaſures of length, as feet, inches, parts of an inch, &c. the angles are eſtimated by arcs of circles conſiſting of degrees, minutes, &c. and, for convenience, theſe

circular meaſures are repreſented by right lines called chords, right-ſines, tangents, &c.

DEF. II.

A CHORD is a right line drawn from one end of an arc to the other: thus A I (fig. 1.) is the chord of the arc A H I, and of A B K F I.

DEF. III.

The RIGHT-SINE, or (as it is commonly called) the SINE, of an arc or angle is a right line drawn from one end of the arc, perpendicular to a radius drawn thro' its other end:

Or, it is half the chord of an arc which is double to that of which it is the ſine.

Thus A N is the ſine of the arcs A H and A B K F; or the angles A C H and A C F.

DEF. IV.

The TANGENT of an arc is a right line, which, touching one end of the arc, (i. e. being perpendicular to a radius drawn to that point of the arc) is continued till it meets a right line drawn from the center thro' the other end of the arc; which laſt line is called the SECANT of that arc.

Thus H G is the tangent, and C G the ſecant of the arc A H, or the angle A C H; F M is the tangent, and C M the ſecant, of the arc A B K F, or the angle A C F.

DEF. V.

A VERSED-SINE, or VERSE-SINE, is that part of the radius which is intercepted between the arc and

and its ſine: thus N H is the verſe-ſine of the arc A H, or the angle A C H.

DEF. VI.

The COMPLEMENT of an arc is what it wants of 90°; and the ſine, tangent, ſecant and verſe-ſine of the complemental arc, are called the CO-SINE, CO-TANGENT, CO-SECANT and CO-VERSE-SINE of the arc of which it is the complement: thus, if H A B be = 90°,

A B is the complement
E A is the co-ſine
B D is the co-tangent
C D is the co-ſecant
E B is the co-verſe-ſine

} of the arc H A, or the angle H C A.

DEF. VII.

The SUPPLEMENT of an arc is what it wants of 180°: thus, ſuppoſing H A B K F be = 180°, A B K F is the ſupplement of A H; and A H of A B K F.

SCHOL. I.

Since A N is the ſine both of the arc A H and A B K F (def. III.) and as this would have been the caſe wherever A had been taken, it follows that the ſame ſine will ſerve for any two angles, of which one is as much greater, as the other is leſs, than 90°; v. g. 80° and 100°; 60° and 120°; or, in other words, every arc and its ſupplement have the ſame ſine.

SCHOL. II.

The ſame tangent and ſecant will ſerve for any arc and its ſupplement, v. g. for A H and A B K F, (fig. ib.) For A C H = F C M (*vertical angles*) C H G = C F M (*right angles*) and C H = C F (*radii*); therefore (by Eucl. I. 26) the triangle F C M = H C G, and the correſpondent ſides are equal; v. g. F M and M C, the tangent and ſecant of A B F (vid. def. IV.) are = H G and G C, the tangent and ſecant of A H.

SCHOL. III.

The largeſt ſines, tangents and ſecants are of 90? in any calculations, therefore, where an obtuſe angle is concerned, the ſine, tangent, or ſecant of its ſupplement are uſed.

SCHOL. IV.

The co-ſine of an arc is always equal to that part of the radius which is intercepted between the center and the ſine; or it is = rad. — verſe-ſine; thus C N is = E A, and is often called, the co-ſine of A H.

AN EASY

INTRODUCTION

TO PLANE

TRIGONOMETRY.

PART II.

Of the triangular canon.

DEF. VIII.

THE TRIANGULAR CANON is a table, in which each degree and minute in an arc of 90° is inſerted, and againſt it the length of its ſine, tangent and ſecant. N. B. The radius is ſuppoſed to be divided into a certain number of equal parts (commonly 100000), and the length of the ſine, tangent, or ſecant of each degree and minute is eſtimated by the number of ſuch parts contained in it.

CHAP. I.

Of forming the triangular canon MECHANICALLY.

PROP. I.

To meaſure the chords, ſines, tangents and ſecants of every 10th degree of a quadrant mechanically.

SOL.

1. Let A B (fig. 2.) be the radius; make A C and B D perpendiculars to A B; on A, with the radius A B, deſcribe the arc B E, which will be that of a quadrant; divide the arc B E into nine equal parts, each of which will conſiſt of 10°; mark theſe diviſions, 10, 20, &c.

2. Draw the line B E; and placing one foot of the compaſſes in B and the other ſucceſſively in the points of the arc B E marked 10, 20, 30, &c. lay thoſe intervals on the line B E towards E, and number the diviſions as in the figure, 10, 20, 30, &c: B E will then be a line of chords, and the diſtance from B to any number will be the chord of an arc of ſo many degrees as the number expreſſes, in a circle whoſe radius is A B (R).

3. Thro' the ſeveral diviſions of the arc B E draw lines to A B, parallel to A E, and they will be perpendicular to A B. Theſe will be the ſines of their reſpective arcs, and A B will be a line of ſines, beginning at A; A F being = the ſine of 10°, A G of 20°, &c.

4. If a ruler be laid on A and the ſeveral diviſions of the arc B E, and lines be drawn to B D, they will cut it in the points marked 10, 20, 30, &c: the diſtances from B to 10, 20, 30, &c. in the line B D, will be the tangents of thoſe ſeveral arcs.

5. From

5. From A to 10, 20, 30, &c. in the line of tangents is the length of the ſecants arcs of 10°, 20°, 30°, &c. With one foot of the compaſſes in A, and the other in the ſeveral points of the tangent line marked 10, 20, 30, &c. draw arcs to the line A C; A C will then become a line of ſecants.

6. Having a line of equal parts, of which ſo much as is equal to radius ſhall contain an even number, v. g. 100, 1000, 10000, &c. the length of any ſine, tangent or ſecant being applied to that line, its proportion to radius may be found; and, from thence, its abſolute length, if that of the radius is known.

SCHOL.

If the arc B E were divided into 90 equal parts, and each of theſe into 60, the ſines, tangents and ſecants of every minute of a degree might be drawn, and the proportion of each to the radius be found, as above: but ſince they can neither be made nor meaſured with ſufficient accuracy, recourſe muſt be had to calculation, the method of which is laid down in the next chapter.

CHAP. II.

Of forming the triangular canon by CALCULATION.

Leſt the connexion of the following problems which relate to this matter ſhould be broken, it may be proper here to lay down and demonſtrate the following theorem, the uſe of which will ſoon appear.

PROP. II.
A THEOREM.

If three arcs B C, B D, B E, (fig. 3.) be in arithmetical proportion, then as the radius A D is to twice the co-sine of their common difference C D, so is D H the sine of the mean to C I + E F the sum of the sines of the two extremes.

CONSTRUCTION.

Draw E C cutting A D in O, and O G parallel to D H.

DEMONSTRATION.

	1. E D being (by hyp.) = D C, E O must be = O C; and therefore A O perpendicular to E C (Eucl. III. 3.)
1.	2. O C is the sine, and A O the co-sine, of D C, the common difference.
1.	3. E O being = O C, O G is an arithmetical mean between C I and E F, and consequently 2 O G = C I + E F.
Conſ.	4. A D : A O :: D H : O G
4.	5. A D : 2 A O :: D H : 2 O G
5. 3.	6. A D : 2 A O :: D H : C I + E F.
6. 2.	7. Radius : twice cos. of dif. :: sine of the mean : sum of the sines of the extremes Q. E. D.

PROP. III.

Supposing the radius divided into 100000 equal parts, to find how many such parts the sine of 1′ will contain.

LEMMA

LEMMA I.

As there is little difference between ſo ſmall an arc and its ſine, the meaſure of the arc may be taken for that of the ſine.

LEMMA II.

The diameter of a circle is to the periphery very nearly as 113 is to 355.

SOL. AND DEM.

1. As the diameter is to the periphery, ſo are the parts contained in the diameter, to thoſe in the periphery. The number of parts therefore in the periphery may be had by lemma 2, and the common rule of three: thus, 113 : 355 :: 200000 : 628318.58.

2. Divide the parts of which the periphery conſiſts by 21600, the number of minutes in a circle; the quotient is the number of parts contained in an arc of 1′. and by lem. 1, in its ſine: thus $628318 \div 21600 = 29.08882$ the ſine of 1′.

PROP. IV.

The ſine of an arc, as D G (fig. 4.) being given; to find its co-ſine.

SOL. AND DEM.

Since D G A is (by hyp.) a right angle, (by Eucl. I. 47) $A D^2$ is $= D G^2 + A G^2$; take therefore $D G^2$ from $A D^2$, the ſquare root of the remainder will be A G, $=$ C D, the co ſine of D B.

SCHOL.

SCHOL. I.

Suppoſe D G to be the fine of 1′, or by prop. III. 29.08882;

then from— 10000000000. the ſquare of A D, or radius,
take ——————— 846.1594 the ſquare of D G, there will
remain ——— 9999999153.8406, the ſquare of A G or C D.

The ſquare root of this, (99999.99577,) is the co-fine of an arc of 1′, in parts of which the radius contains 100000.

SCHOL. II.

Hence the verſe-fine, being A B — A G, or radius — co-fine; may eaſily be found; in the given caſe from the radius 100000.

take the co-fine 99999.99577

there will remain ——,00423, the verſe-fine of an arc of 1′.

SCHOL. III.

The co-verſe-fine may be found in the like manner: for, ſince it is A F = AC, and AC is = D G, it is radius — the fine.

from the radius 100000.

take ——————— 29.08882 the fine of 1′.

there will remain 99970.91118, the co-verſe-fine of B D, an arc of 1′.

PROP. V.

To find the fine of 2′.

SOL. AND DEM.

1. Take the fine and co-fine of 1′, as found by prop. III. and ſch. I. of prop. 4.

2. Then,

2. Then, since the arcs of 0′, 1′ and 2′ are in arithmetical proportion, and the common difference is 1′, it follows from the theorem prop. II. that as the radius is to double the co-sine of 1′, so is the sine of 1′ to the sum of the sines of 0′ and 2′.

SCHOL.

The sine of 1′ being 29.08882, and the co-sine 99999.99577, the double of which is 199999.99154, the proportion is

100000 : 199999.99154 :: 29.08882 : 58.177637539

the sum of the sines of an arc of 0′ and 2′; and as the sine of 0′ is 0, this will be the sine of an arc of 2′.

PROP. VI.

To find the sine of an arc of 3′.

1. Take the sine of 2′, as found by prop. V. and the co-sine of 1′ by sch. I. of prop. IV.

2. Then, since the arcs of 1′, 2′ and 3′ are in arithmetical proportion, and the common difference is 1′; it follows from the theorem, prop. II. that as the radius is to double the co sine of 1′, so is the sine of 2′ to the sum of the sines of 1′ and 3′.

3. Substract the sine of 1′ from the 4th proportional, the remainder will be the sine of 3′.

SCHOL. 1.

The operation will stand thus;

100000 : 199999.99154 :: 58.177637539 : 116.355270156171886

Substract the sine of 1′. ——— 29.08882

there will remain for the sine of 3′ ——— 87.266450156171886

SCHOL.

SCHOL. II.

All the ſines in a regular ſeries up to 1°, and from thence to 90° may be found in this manner.

SCHOL. III.

Theſe operations may be greatly ſhortened by neglecting moſt of the decimal places. Where the radius is 100000, two decimal places are ſufficient; but if the third decimal place has a figure of higher value than 5, one is uſually added to the ſecond: thus, the ſine of 1′ is 29.09; of 2′ is 58.18; of 3′ is 87.27.

SCHOL. IV.

There are many other methods of finding the ſines of the ſeveral arcs in a quadrant; but this may ſuffice to give ſome notion of the manner of finding them, and of the nature of meaſuring them. As to practice, the tables in printed books may be depended upon, as ſufficiently exact.

PROP. VII.

The ſine and co-ſine of any arc, or angle, being given, to find the tangent, ſecant, co-tangent and co-ſecant of it.

SOL. AND DEM.

Since D G and H B, (fig. 4.) are by hypotheſis perpendiculars to A B, the triangle A G D is ſimi-

lar

lar to A B H; and *par. rat.* the triangle A C D is similar to A F E: therefore (by Eucl. VI. 4.)

1. A G : G D :: A B : B H
   Co-sine : Sine :: Radius : Tangent.

2. A G : A B :: A D : A H
   Co-sine : Radius :: Radius : Secant.

3. A C : A D :: A F : A E
   Sine : Radius :: Radius : Co-secant.

4. A C : C D :: A F : F E
   Sine : Co-sine :: Radius : Co-tangent.

SCHOL.

The triangle A B H is similar to A F E, (for A B H = A F E, *right angles;* F E H = H A B, alternate angles, as F E is parallel to A B; and F A E = A H B, for the like reason:) therefore

H B : B A :: A F : F E
Tangent : Radius :: Radius : Co-tangent.

Hence it follows that when the tangent is equal to the radius, the co-tangent will also be equal to it, and consequently, to the tangent: now the tangent of no angle but one of 45° can be equal to its co-tangent; and therefore the tangent of an angle of 45° is equal to the radius, or to a sine of 90°, or a chord of 60°.

CHAP.

CHAP. III.

Of the printed triangular tables.

DEF. IX.

The printed tables contain all the gradations by which arcs aſcend from 1′ to 90°, each arc being 1′ larger than the preceding, and oppoſite to each are placed its ſine, tangent, co-ſine and co-tangent, and ſometimes the ſecant and co-ſecant.

DEF. X.

Some of theſe tables conſider the radius as conſiſting of 100000 equal parts, and give the length of the ſines, tangents, &c. in ſuch parts: theſe are called the tables of natural ſines, tangents, &c.

Other tables conſider the radius as conſiſting of 10000000000 equal parts, and expreſs the length of the ſines, tangents, &c. by the logarithms of that number, the radius being 10. Theſe are, therefore, called the tables of logarithmic, or artificial ſines, tangents, &c.

SCHOL. I.

As to the co-ſine, co-tangent and co-ſecant of any arc are the ſine, tangent and ſecant of its complement, (def. VI.) if two arcs which are the complements of each other are placed on oppoſite pages, the ſine, tangent and ſecant of each will be the co-ſine, co-tangent and co-ſecant of the other.

Thus

Thus in ſome tables all the gradations from 1′ to 45°, with their ſines, &c. are placed on the left hand pages, and the minutes are reckoned from the top: but the degrees from 46 to 90 are placed on the right hand pages: are reckoned backwards, i. e. from the end of the book; and the minutes from the bottom to the top of the page. By this contrivance each gradation ſtands againſt its complement, and the ſine, &c. of each is the co-ſine, &c. of its oppoſite.

In other tables the degrees from 1 to 45 are placed at the top of the page, reckoning forwards, and the minutes ſtand at the left ſide of the columns, increaſing from the top to the bottom: but the degrees from 46 to 90 are placed at the bottom of the page, and are to be reckoned backwards, and the minutes ſtand on the right ſide of the columns, increaſing from the bottom to the top: the ſine, &c. of the degree ſet at the top, are the co-ſine, &c. of that at the bottom, and v. v.

SCHOL. II.

If an angle be given in degrees and minutes, and the ſine, &c, are required; find the degree on the top or bottom of the page, the ſine, &c. are in the line oppoſite to the minute given, v. g. what are the logarithmic ſine and tangent of an angle of 6°. 12′? *Anſ.* The ſine 9.03342; the tangent 9.03597:

Of an angle of 78°. 22′? *Anſ.* The ſine 9.99099; the tangent 10.68639.

If the ſine, &c. be given, and the degree and minute are deſired; find the neareſt number to that

that given among the fines, &c. the minute at the end of the line, and the degree at the top or bottom of the page, give the anſwer. v. g. What is the angle of which 9.27602 is the ſine? *Anſ.* 10°. 53′—or of which 10.46720 is the tangent? *Anſ.* 71°. 10′, for that is the tangent of 10.46715, which is the neareſt to the given number.

The logarithm of a given number is found over againſt it; v. g. what is the logarithm of 214? *Anſ.* 2.33041——of 214.4? *Anſ.* 2.33122——of 2144? *Anſ.* 3.33122.

The logarithm being given, the number is found oppoſite to that logarithmic number which is neareſt to it, v. g 3.93959 being given, 3.93957 is the neareſt, which is the logarithm of 8701.

N. B. The logarithm of any number will, with a different index, ſerve for another which is 10 for 100 times greater. Thus 1.44731 is the log. of 28.01——2.44731 of 280.1——3.44731 of 2801. Hence, to ſhorten the table, the indexes are often omitted; but they may eaſily be ſupplied, for they are always one leſs in *value* than the *number of figures* in their natural numbers: i. e. from 1 to 10 the index is 0; from 10 to 100, it is 1; from 100 to 1000, 2; from 1000 to 10000, 3.

SCHOL. III.

Moſt trigonometrical calculations are made by the rule of three; and as the natural ſines, tangents, &c. are large numbers, calculations made by them muſt be tedious: but ſince numbers are multiplied by the addition of their logarithms, and diviſion is performed by ſubſtracting the logarithm

of

of the diviſor from that of the dividend, to work a ſum in the rule of three is only to add the logarithms of the 2d and 3d, and ſubſtract the firſt from their ſum. The artificial ſines, tangents, &c. are therefore the moſt eaſy and convenient.

SCHOL. IV.

A ſtill ſhorter method of working queſtions in the rule of three, than that mentioned in the laſt ſcholium, has been invented; which is, by uſing what is called co. ar. or complement arithmetic. The radius is reckoned 10, ſubſtract any given ſine, &c. from 10, the remainder is co. ar.

N. B. If every figure, beginning at the left hand be ſubſtracted from 9, except the laſt, and this be taken from 10, the operation will be eaſier and the anſwer the ſame.

Now it is plain from ſch. II. that the ſum of the logarithms of the 2d and 3d proportionals exceeds the logarithm of the fourth by the value of the firſt, becauſe, when the firſt is ſubſtracted the remainder is the 4th: now if inſtead of ſubſtracting the firſt, the co. ar. be added, the ſum will be too large by the whole radius. Hence,

Inſtead of the logarithm of the firſt proportional, put co. ar. add this, and the logarithms of the 2d and 3d; from the ſum ſtrike out radius and the remainder will be the fourth.

A proportion wrought by ſchol. III.

As the fine of an angle ſuppoſe of 35°	9.75859
* is to the fine of another known angle 40°	9.80807
ſo is the ſide oppoſite to the former 380	2.57978
(here add the ſecond and third	12.38785
and ſubſtract the firſt from their ſum)	2.62926

to the ſide oppoſite to the latter 428.8; for that is the number of which 2.62926 is the logarithm.

The ſame proportion wrought by co. ar.

As the fine of an angle of 35° co. ar.	0.24141
is to the fine of an angle of 40° ———	9.80807
ſo is the ſide oppoſite to the former 380	2.57978
(Add all three, dropping radius or 10.)	2.62926

to the ſide oppoſite to the latter 428.8 for that is the number of which 2.62926 is the logarithm.

N. B. The co. ar. of a line is had the ſame way, and will anſwer the ſame purpoſe. The juſtice of the operation depends on much the ſame principles as borrowing and paying in ſubſtraction.

* The ſecond and third proportionals are commonly placed in the contrary order; thus, one fine is to its oppoſite ſide, as another fine is to the ſide oppoſite to it: the nature and reſult of the operation is the ſame; but, as it is moſt natural to compare fine with fine, and ſide with ſide, the above order ſeemed preferable.

AN EASY

INTRODUCTION

TO PLANE

TRIGONOMETRY.

PART III.

A Deſcription of the portable pocket caſe of inſtruments and ſome of the principal uſes of them in trigonometry.

DEF. XI.

THE PROTRACTOR is a ſmall brazen ſemicircle; the mark in the middle of the ſtraight edge, or diameter, is the center of the inſtrument; the ſemicircular edge or limb is divided into degrees, which are numbered from each end.

SCHOL. I.

To meaſure a given angle by the protractor, apply the diameter to one ſide, and the center to the angular point; the degree of the limb under which the other ſide paſſeth is the meaſure of the angle.

SCHOL. II.

To lay down an angle of any given number of degrees by the protractor, draw a line for one ſide of the angle; apply the diameter to it, and the center to the point where the angle is to be made; mark the point under the given degree, and draw the other ſide thro' it.

SCHOL. III.

To raiſe a perpendicular from a given point in a line; v. g. C, in A B (fig. 5.) place the center of the protractor on C, and the 90° on the line A C, or B C, a line drawn along the ſtraight edge, will be the perpendicular required.

To let fall a perpendicular from a given point (as F fig. ib.); lay the center and 90° on the line, and ſlide it backward or forward till the ſtraight edge touches the given point.

DEF. XII.

The IVORY RULE is a ſcale of equal parts; and on one ſide is a ſcale of chords, marked C.

SCHOL. I.

On one ſide of the ivory rule is a ſcale of 6 inches divided into 60 and 50 * parts, and below it a ſcale of $\frac{1}{2}$ and $\frac{1}{4}$ of an inch divided into 100 parts; but, as this is chiefly uſed in ſurveying, it will be deſcribed hereafter. Prop. XXV. gr. 4.

* The diviſion into 50 parts is particularly adapted to the meaſuring of glaſs and ſome other things, for which a foot rule divided into 100 parts is commonly uſed.

On

On the other ſide, one inch is divided into 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5 and 6 equal parts, and one of theſe parts being ſubdivided into 10, each may be conceived to conſiſt of the ſame number, and thus the ſcales aſſiſt in dividing an inch into 30, 35, 40, &c. equal parts.

If each larger diviſion be reckoned 1, each of the ſmaller will expreſs $\frac{1}{10}$ part of 1; if the larger ſtand for tens, the ſmall ones will denote units; and if the larger expreſs hundreds, each ſmall one will be 10. v. g. place one foot of the compaſſes on the 4th large diviſion, and the other on the 8th of the ſmall ones; the diſtance may either expreſs 4.8, or 48, or 480, as the value of 4 is ſimply 4, or 40, or 400.

The uſe of theſe ſcales for *drawing or meaſuring figures* is obvious. Suppoſe a field be a parallelogram of 76 × 89 yards; to delineate this on paper, take 76 from one of the ſcales, according as the plan is to be large or ſmall, and draw a line of this length; then take 89 from the ſame ſcale, and draw a line of this length at right angles with the former; compleat the figure. It will be ſimilar to the field, and the diagonal of the field may be found by taking the diagonal of the figure and applying it to the ſame ſcale, the yards of one anſwering to the number of equal parts in the other.

A perpendicular on the end of a line, as B (fig. 6.) may eaſily be made by the ſcales; take B D = 4 parts, then on B with 3, and D with 5 of the ſame parts, ſweep arcs interſecting in E; E B will

be perpendicular to A B, for $5^2 = 3^2 + 4^2$, and therefore E B A is a right angle (by Eucl. I. 48.)

The diviſions at the left end of theſe ſcales are likewiſe ſubdivided into 12 equal parts, for the convenience of drawing plans of buildings, &c. the larger diviſions repreſenting feet, and the ſmaller inches.

SCHOL. II.

To meaſure an angle by the ſcale of chords; as D B A (fig. 7.) take in the compaſſes the chord of 60°: (i. e. place one foot in o the other in 60°;) with this interval, on the angular point B as a center, ſweep an arc C H from one ſide to the other; the diſtance between the points C and H where the arc cuts the ſides applied to the ſcale of chords (always remembering to ſet one foot of the compaſſes at o) will incloſe ſo many degrees as the angle conſiſts of.

To lay down a given angle; v. g. 45° draw one ſide as A B (fig. ib.) mark B the point where the angle is to be; with that point as a center, and a chord of 60° for the radius, ſweep an arc C E cutting the ſide already drawn in C, take from the ſcale of chords 45 the number of degrees of which the angle is to conſiſt; one foot of the compaſſes being placed in C the point where the arc cuts the ſide already drawn, the other will reach along the arc to H, the point thro' which the other ſide muſt be drawn.

It is eaſy to make *a perpendicular at the end of a given line*, as at B (fig. 7.) by the ſcale of chords; on B, with the chord of 60° for a radius, ſweep an

an arc; from C lay off the chord of 90°, which reaches to E, thence draw the perpendicular, E B.

But there is little need to mention this method, or that propoſed in the laſt ſcholium, ſince it is ſo eaſy to perform this by the compaſſes only; thus, upon a center at any convenient diſtance above the line, as C (fig. 8.) with the radius C B, deſcribe a circle cutting A B in D; a ruler laid over D and C will cut the circle in E, the point from whence the perpendicular is to fall upon B. (compare Eucl. III. 31.)

The greateſt chord is of 180°; but the ſcale reaches no farther than 90°, becauſe the chord of an angle of more than 90° may eaſily be taken at twice from thoſe under 90: v. g. 50° and 60° for 110°; and were the ſcale continued, the higher degrees would be ſo ſmall they could not be diſtinguiſhed, unleſs the ſcale were made of an inconvenient length.

DEF. XIII.

The SECTOR is a folding rule, generally 6 inches long when it is ſhut: it contains ſeveral ſcales, of which ſome are ſingle and others double.

Each *ſingle ſcale* commonly conſiſts of three parallel lines properly graduated by tranſverſe lines.

Each *double ſcale* conſiſts of a pair of ſingle ſcales proceeding from the center as radii. The moſt common way of meaſuring by the double ſcale is to take the diſtance a-croſs the inſtrument between one degree and its fellow. v. g. from 30 to 30: and this is called the *tranſverſe diſtance* of ſuch a degree. *N.B.* The compaſſes muſt be ſet

on the line of each ſcale, which is neareſt the inner edge of the ſector. But ſometimes the meaſure is to be taken from the center along one leg, as tho' it were a ſingle ſcale, to ſome given degree, and this is called a *lateral diſtance* of ſuch a degree.

DEF. XIV.

Along one ſide of the ſector, when open, is a common ſcale of 12 inches, each divided into 10 parts. Below the former half of this ſcale, when the ſector is ſhut, is, a ſingle ſcale of the inclination of meridians, marked I M; of chords, marked C; and, at the bottom, of latitudes, marked L; and of hours, marked H. In the middle is a double ſcale of chords, marked C C; of ſecants, marked ſ. ſ; of lines, marked L; and a line of polygons, marked P.

SCHOL. I.

The ſcale of the *inclination of meridians* is chiefly uſed in ſpherical trigonometry; the uſe of the ſcales of *latitude and hours* will appear by prop. XXXVII.

SCHOL. II.

The *ſingle ſcale of chords* is the ſame as was deſcribed def. XII. ſch. II.

SCHOL. III.

The ſingle ſcale of chords can only be uſed when the radius is the chord of 60° upon the ſcale. But *the double ſcale of chords* may always be uſed, unleſs the radius is ſhorter than the diſtance from 60°

60° to 60° when the inſtrument is ſhut, or longer than that diſtance when it is open. The tranſverſe diſtance of any degree is the length of it's chord, in a circle whoſe radius is the tranſverſe diſtance of 60°. The ſector, therefore, muſt always be open till the diſtance from 60° to 60° is = radius.

The double ſcale riſes no higher than 60°; if a larger angle is to be made or meaſured, the chord of a part muſt be taken, and then of the remainder (as of 60° and 30°, for 90°.) or rather of $\frac{1}{2}$ or $\frac{1}{3}$, (as 45° twice, or 30° three times, for 90°) becauſe in this way the compaſſes need to be ſet but once.

To make a very ſmall angle, v. g. 4°, firſt lay off the chord of 60°, then meaſure back 56, the remainder is the chord of 4°.

N. B. Each leg of the double ſcale may be uſed ſingly as a ſcale of chords.

SCHOL. IV.

The *ſecants* are meaſured the ſame way as chords, only the beginning of the ſcale (o and o) muſt be ſet the length of radius aſunder, becauſe the leaſt ſecant is greater than the radius.

SCHOL. V.

The *double ſcale of lines* conſiſts of two ſingle ſcales of equal parts, each divided into 10, and theſe parts again into 10, and each of theſe ſmaller parts into 2. If the larger diviſions are units, the ſub-diviſions will be tenths and their halves $\frac{5}{100}$ parts; if the larger are tens, the ſmaller will

will be units and their halves $\frac{5}{10}$; if the larger are hundreds, the smaller will be tens, and each of their halves 5. This scale will assist us

1. *To divide a given line into any number of equal parts.* v. g. into 8; make the whole line a transverse distance of 8, the transverse distance of 1 will be $\frac{1}{8}$, of 2 $\frac{2}{8}$, &c. *N.B.* In this, and in all such cases, it is better to take the several distances of 1, 2, 3, &c. successively from the scale, than to repeat the measure of one; for in applying a measure several times, if but a small error is made in taking the distance, it is multiplied and becomes sensible.

2. *To diminish a line in any given proportion*; v. g. a line of 4 inches in the proportion of 8 to 7, make the lateral distance of 7 the transverse of 8; mark how far 4 inches will reach from the center, the transverse distance to the correspondent point is the line required *.

3. *To find a third proportional to two given quantities*; v. g. 2 and 6; take the lateral distance of the second term (6), and make it a transverse distance of the first (2); the transverse distance of the second (6) applied laterally will give the 3d (18). *N.B.* If the sector will not admit the lateral distance of the second to become a transverse distance of the first, take $\frac{1}{20}$ $\frac{1}{30}$ $\frac{1}{4}$; but remember to multiply the last term by the denominator. Thus in the given instance, make $\frac{6}{2}$, i. e. 3, the transverse distance of 2; then the transverse distance of 6, applied laterally will reach 9, which multi-

* Thus plans, maps, pictures, &c. may be reduced in any desired proportion.

plied

plied into 2 gives 18; thus, $6 : 2 : \frac{2}{3}$. This method may be uſed in the next and many other ſimilar caſes.

4. *To find a 4th proportional to 3 given quantities*, 3, 4, 6; take the lateral diſtance of the ſecond (4) and make it the tranſverſe diſtance of the firſt (3); the tranſverſe diſtance of the 3d (6) applied laterally will give the 4th term (8); thus, $10 : 7 :: 3 : 2\frac{1}{10}$.

5. *To find a mean proportional to 2 given quantities*, v. g. 40 and 90; ſet the ſcales of lines at right angles, (by taking the lateral diſtance of 5, and opening the ſector till this will reach a-croſs from 4 to 3, for $5^2 = 4^2 + 3^2$ and therefore the angle is right by Eucl. I. 48.) then, finding half the ſum of the given lines, $\left(\frac{90 + 40}{2} = 65\right)$ and their half difference, $\frac{90 - 40}{2} = 25$, and taking in the compaſſes the lateral diſtance of the half ſum (65), place one foot in the half difference (25), the other will reach a-croſs the ſector to the mean proportional, (60).

SCHOL. VI.

To make a polygon in a given circle by the ſcale of polygons; the radius being made a tranſverſe diſtance of 6, the tranſverſe diſtance of 4, 5, 7, &c. will be the ſide of a polygon of 4, 5, 7, &c. ſides; apply this diſtance round the circle.

To make a polygon on a given line, as A B (fig. 9.) make the line A B the tranſverſe diſtance of 5 for

a

a pentagon, of 7 for a heptagon, &c. with the tranſverſe diſtance of 6 as a radius, upon each end of the given line A and B, deſcribe arcs cutting each other in C, and from the point of interſection, with the ſame radius deſcribe a circle, which will paſs thro' the ends of the given line; the given line, applied round the circle, will form the polygon required.

The ſcale of polygons may be uſed as a ſingle ſcale; thus, with the lateral diſtance of 6, deſcribe a circle, the lateral diſtance of any other number, applied round the circle, will point out the ſides of a polygon of ſo many ſides.

Polygons may be made by the ſcale of chords, either ſingle or double; thus, make a circle with the chord of 60° for its radius, divide 360 by the number of ſides required, take a chord equal to the quotient, and apply this interval round the circle: thus $360 \div 5 = 72$, which is therefore the ſide of a pentagon.

Polygons may be made on a given line by the protractor, or any inſtrument for laying down an angle; thus,

Let A B, (fig. 10.) be the given line: divide 360 by the number of ſides, the quotient is the angle at the center; ſubſtract this from 180°, the remainder is the angle at the circumference: at A and B draw A C and B D which ſhall make C A B and D B A, the proper angles at the circumference; make A C and B D = A B, and thus proceed till the polygon is compleat.

Or thus; from the ends of the given line A B (fig. 11.) draw A C and B D, making C A B and D B A

D B A equal to half the proper angles at the circumference of the given polygon: the line A C and B D continued, will interſect in E; on E as a center, with E A or E B as a radius, deſcribe a circle, and round it apply the given line A B.

D E F. XV.

Along the upper edge of the other ſide of the ſector, when open, is a ſingle ſcale of numbers marked N—, of ſines, marked S, and of tangents marked T. In the middle of the inſtrument, when ſhut, is a double ſcale of ſines marked S S, and two of tangents, marked T T, and t t.

S C H O L. I.

In the *ſcale of numbers*, 1 is placed at the beginning; 10 at the middle, and at the upper end.

1 may ſtand for $\frac{1}{10}$, 1, 10, 100, &c; now whatever be its value, the firſt 10 will be ten times ſo much, or contain one place of figures more, and the ſecond 10 will be 100 times the value of 1, or contain two more places of figures. Thus,

If 1 be $\frac{1}{10}$, the firſt 10 will be $\frac{1}{10}$, the ſecond 10 will be 1.

$\frac{1}{10}$	——	1	——	10.
1	——	10	——	100.
10	——	100	——	1000.
100	——	1000	——	10000.

and the intermediate diviſions muſt be eſtimated agreeable hereto.

This

This ſcale is of logarithmic numbers, i. e. the diſtance expreſſes the ratios of one to another, v. g. the diſtance of 2 from 1 is equal to that from 2 to another which has the ſame ratio to 2, as 2 has to 1, i. e. 4.

Hence we learn 1) *to multiply:* for ſince 1 has the ſame ratio to either of the factors, as the other has to the rectangle; the diſtance from 1 to the multiplier will reach from the multiplicand to the rectangle, v. g. the diſtance from 1 to 3 will reach from 4 to 12.

2. *To divide:* for, ſince the diviſor is to 1 as the dividend to the quotient, the diſtance from the diviſor to 1 will reach from the dividend to the quotient: thus the diſtance from 4 to 1 will reach from 12 to 3.

3. *To work the rule of three:* for, ſince the firſt is to the ſecond as the third is to the fourth, the diſtance from the firſt to the ſecond will reach from the third to the fourth, only remembering that if the ſecond be leſs than the firſt, the diſtance from the third muſt be ſet off backward to find the fourth, v. g. 3 : 6 :: 9 : 18, or 6 : 3 :: 18 : 9.

A few examples may be proper.

Find a ſeries of *continued proportionals* to the two numbers 1 and 2, or 1 and $1\frac{1}{2}$. *N.B.* This is an eaſy method of trying whether the ſcale be truly divided.

Queſtions in multiplication; $6 \times 4 = 24$, $40 \times 3 = 120$; $35 \times 24 = 840$; $375 \times 60 = 22500$.

Queſtions in diviſion; $36 \div 4 = 9$; $144 \div 9 = 16$; $1728 \div 12 = 144$.

Queſtions

Queſtions in the rule of three; 3 : 8 :: 15 : 40; 5 : 12 :: 38 : $91\frac{1}{5}$; 18 : 4 :: 364 : $80\frac{8}{9}$.

But where a queſtion, or the reſult of it, runs to more than 3 places, it may more exactly be ſolved by the pen, as we can do little better by the beſt inſtruments than gueſs at the value of the loweſt of 4 or more places.

SCHOL. II.

The ſingle ſcales of ſines and tangents are likewiſe logarithmic; but their uſe cannot well be ſhewn till the ſolution of triangles has been explained.

SCHOL. III.

The double ſcale of ſines being opened till the radius is a tranſverſe diſtance of 90°, the tranſverſe diſtance of any other degree is the length of the ſine of ſuch a degree in a circle of that radius.

SCHOL. IV.

The tranſverſe diſtance of 45° on the *double ſcale of tangents* being made equal to the radius, the tranſverſe diſtance of any other degree is the length of the tangent of that degree, in a circle of that radius. If the tangent in queſtion be of fewer than 45°, the ſcale marked T T is to be uſed; if of more, the ſcale marked t t.

N. B. The greateſt tangent and ſecant are thoſe of 89°. 59′; for the ſecant of 90 is parallel to the tangent; and, conſequently both may be produced infinitely without meeting. Even the tangents of more than 75° are too long for uſe, and therefore the

the ſcales are continued no farther; and the cotangents are frequently uſed inſtead of the tangents of more than 45°.

SCHOL. V.

The diſpoſition of theſe double ſcales upon the ſector is ſuch, that the tranſverſe diſtance between the extremities of the chords, ſines and lower tangents is equal; ſo that when any one of them is adjuſted to a circle of a given radius, all the reſt will be ſo too. Thus, if the radius of a circle were two inches, and the chord, ſine or tangent of any angle, v. g. 28°. 30′. were deſired; make two inches the tranſverſe diſtance either of 60° of the chords, 90° of the ſines, or 45° of the tangents, the tranſverſe diſtance of 28°. 30′. may be taken on the ſcale of chords, ſines or tangents, without altering the ſector.

But if a tangent of more than 45° be wanted, two inches muſt be made the tranſverſe diſtance of the tangents marked t t.

To find the ſecant of 28°. 30′, make two inches the tranſverſe diſtance of 0, and that of 28°. 30′ will be the ſecant deſired.

AN EASY

INTRODUCTION

TO PLANE

TRIGONOMETRY.

PART IV.

Of the ſolution of plane triangles.

DEF. XVI.

TO ſolve a triangle is to find ſome of the angles and ſides, others being given.

SCHOL. I.

As the angles of two triangles may be equal, when the ſides are not ſo, nothing can be judged of the ſize of a triangle merely by knowing its angles: one ſide, therefore, at leaſt muſt be given; but the reſt may be found, if

1. Two angles be given, and a ſide oppoſite to one of them: or, two ſides, and an angle oppoſite to one of them: or,

2. Two ſides and the angle included between them: or

3. All the ſides.

SCHOL. II.

Triangles may be ſolved either by protraction, or calculation. If the three things mentioned in either of the caſes, ſch. I. are given, a line may be made proportionable to each given line, by the ſcale of equal parts; and an angle equal to each given angle, by the protactor, ſcale of chords, &c. and thus the triangle being compleated, the length of the unknown lines may be known by being applied to the ſame ſcale of equal parts, and the angles by the protractor, ſcale of chords, &c. This is protraction; but in many caſes it is neceſſary to be more exact, and therefore, recourſe muſt be had to calculation, the rules of which are laid down below; but as ſome of them depend on the two following theorems, it may be proper to demonſtrate them before we proceed.

PROP. VIII.

THEOREM.

In all plane triangles the ſides are to each other in the ſame proportion as the ſines of the angles oppoſite to them: v. g. (fig. 12.)

A B : A C :: ſine of A C B : ſine of A B C.

CONSTRUCTION.

Take C E = A B, and draw E F and A D perpendiculars to C D.

De-

DEMONSTRATION.

Eucl. VI. 4.	1. Since the triangles E C F and A C D are ſimilar E C : A C :: E F : A D.
Cons.	2. E C = A B.
1, 2.	3. A B : A C :: E F : A D.
	4. E F is the ſine of A C B and A D of A B C to the ſame radius E C or A B : therefore
3, 4.	5. A B : A C :: the ſine of A C B : the ſine of A B C. Q. E. D.

PROP. IX.

THEOREM.

Of any two unequal quantities,

1. The half difference added to their half ſum is = the greater.

2. The half difference ſubſtracted from the half ſum is = the leſs.

3. The half ſum taken from the greater will leave half the difference of the quantities.

CONSTRUCTION.

Let A B (fig. 13.) be the greater, and B C the leſs; take A D = B C, and biſect D B in E.

DEMONSTRATION.

Cons.	1. D E and E B are each equal to half the difference.
1. Cons.	2. A D + D E = B C + B E, i. e. A E = half the ſum.
1, 2.	3. A E + B E = A B: i. e. half the ſum with half the difference is = the greater; Q. E. D. 1mo.
1, 2.	4. A E − D E = A D or (by cons.) B C; i. e. half the ſum − half the difference is = the leſs; Q. E. D. 2do.
3.	5. A B − A E = B E; i. e. the greater − half the ſum = half the difference; Q. E. D. 3tio.

PROP. X.

In any plane triangle, if two ſides and the angle oppoſite to one of them, or two angles and the ſide oppoſite to one of them, be given, to find the reſt.

SOL.

Caſe 1. To find an angle, begin with a ſide oppoſite to a known angle; and ſay,

As one known ſide is to another known ſide; ſo is the ſine of the angle oppoſite to the firſt of theſe ſides to the ſine of the angle oppoſite to the latter.

Caſe 2. To find a ſide, begin with an angle oppoſite to a known ſide; and ſay,

As the ſine of one known angle is to the ſine of another known angle; ſo is the ſide oppoſite to the firſt of theſe angles, to the ſide oppoſite the latter.

In

In both cases work the proportion as directed def. X. schol. III. or IV.

EXAMPLE.

Case 1. In the triangle A B C (fig. 14.) let A C = 66, A B = 85 and B = 45°; what is C?

As the side A C 66	——	1.81954
is to the side A B 85	——	1.92942
so is the sine of B 45°	——	9.84948
		11.77890
to the sine of C, i. e. 65°. 36′.		9.95936 65.°

Case 2. Let C (in the same triangle) be = 65°. 36′, B = 45° and A B = 85; what is A C?

As the sine of C = 65°. 36′	co. ar.	0.04064
is to the sine of B = 45°.	——	9.84948
so is the side A B = 85	——	1.92942
to the side A C = 66	——	1.81954

DEMONSTRATION

Is given in prop. VIII.

SCHOL. I.

In the first case; one angle being given, and another found, the third is had by subtracting the sum of them from 180°, (comp. Eucl. I. 32.) and then the remaining side may be found by case 2.

In caſe 2d; two angles being given, their ſum ſubſtracted from 180° gives the 3d: the two unknown ſides may then be found, one after the other, either by the rule laid down in this or the former caſe; and thus all the ſides and angles will be known.

SCHOL. II.

This problem may be ſolved by the inſtruments; thus,

I. By the ſingle ſcale of ſines.

Caſe. 1. The diſtance on the ſcale of numbers from (66 = A C) the known ſide oppoſite to the known angle, to (85 = A B) the other known ſide, will reach, on the ſcale of ſines, from (45°) the ſine of the given angle (B) to (65°. 36′) the ſine of the angle (C) oppoſite to that other given ſide (A B).

Caſe. 2. The diſtance, on the ſcale of ſines, from (65°. 36′. = C) the ſine of the known angle oppoſite to the known ſide, to (45° = B) the line of the other known angle, will reach, on the ſcale of numbers, from (85 = A B) the known ſide to (66 = A C) the ſide oppoſite to the other known angle.

II. By the double ſcale of ſines.

Caſe. 1. The known ſide oppoſite to the known angle (A C = 66), taken on a ſcale of equal parts, being made the tranſverſe diſtance of that angle, (B = 45°;) the other given ſide (AB=85) being taken on the ſame ſcale of equal parts, and applied a-croſs the ſector, the degree of which it becomes a tranſverſe diſtance, will be the angle oppoſite to it.

Caſe

Caſe 2. The given ſide oppoſite to a given angle (A B = 85), being taken on a ſcale of equal parts and made the tranſverſe diſtance of (65°. 36′. = C) the ſide oppoſite to it; the tranſverſe diſtance of the other given angle (B = 45°), applied to the ſame ſcale of equal parts, will give (66 = A C) its oppoſite ſide.

PROP. XI.

In any plane triangle, if two ſides and the angle between them be given, to find the reſt.

SOL.

Find the ſum and difference of the known ſides; and ſubſtract half the given angle from 90°, which will leave half the ſum of the unknown angles; then ſay,

As the ſum of the given ſides,
is to the difference of thoſe ſides;
ſo is the tangent of half the ſum of the unknown angles to the tangent of half the difference of thoſe angles.

Having thus got half the difference of the unknown angles, add it to half the ſum for the greater, and ſubſtract it from half the ſum for the leſs.

Thus all the angles being known and two ſides, the third may be found by prop. X.

EXAMPLE.

In the triangle (fig. 15.) A B = 83, A C = 60 and A = 130; what are the angles B and C?

A B 83	83	2) A 130	90°
+A C 60	60	65	½ A 65
the ſum 143.	the diff. 23.	½ of A	25 half the ſum of
of the known ſides.			the unknown angles.

As A B + A C = 143 — co. ar. 7.84466
is to A B − A C = 23 — 1.36173
ſo is the tangent of ½ the unknown angles 25° 9.66867
to the tangent of ½ the difference 4°. 17′ 8.87506

½ ſum 25	½ ſum 25
+½ diff. 4. 17	−½ diff. 4. 17
29. 17 = C	20. 43 = B

DEMONSTRATION.

Conſtruction. On A, with the radius A C, deſcribe a circle, produce B A to E; draw C E and C D; and draw D F parallel to C E. Then

Cons. 1. (Since A C = A E, *radii*) B E is = A B + A C; or it is = ſum of ſides.

Cons. 2. (Since A D = A C, *radii*) B D = A B − A C, or the difference of ſides.

3. 3. (Since A D = A C) A C D = ADC, and conſequently

4.

4. 2 ACD, or 2 ADC = ACD + ADC.
5. ACD + ADC = ACB + ABC (for the triangles ADC and ABC have the angle at A common, therefore (Eucl. I. 32.) the remaining angles of one are = those of the other.

4, 5. 6. 2 ADC = $\overline{\text{ABC} + \text{ACB}}$

6. 7. ADC = $\frac{1}{2}$ $\overline{\text{ABC} + \text{ACB}}$, i. e. $\frac{1}{2}$ the unknown angles.

3, 7. 8. ACD = $\frac{1}{2}$ $\overline{\text{ABC} + \text{ACB}}$.

8. 9. DCB is half the difference of the unknown angles: (for ACD is $\frac{1}{2}$ the unknown angles (gr. 8); now taking this from ACB, which is the greater of them, leaves DCB half the difference of the unknown angles, by prop. IX part III.)

10. (Since DCE, an angle in a semicircle, is a right one) CE is tangent of ADC to the radius DC.

11. DF is tangent of DCB, to the same radius DC: (for DF being by cons. parallel to CE, CDF = DCE *(alternate angles)* and therefore CDF is a right one, comp. gr. 10.)

7, 10. 12. CE is tangent of $\frac{1}{2}$ the unknown angles; and

13. DF

11, 9.	13. DF is tangent of $\frac{1}{2}$ their difference: but
	14. The triangle BFD is ſimilar to BCE; becauſe (by cons.) DF is parallel to EC, (comp. Eucl. VI. 2;) therefore
14. Eucl. VI. 4.	15. BE : BD :: CE : FD.
15, 1, 2, 12, 13.	16. Sum of ſides : differ of ſides :: tangent of $\frac{1}{2}$ ſum : tangent of $\frac{1}{2}$ difference.
	17. The half ſum of the angles being found by the preparation, and the half difference (by gr. 16;) adding the half difference to the half ſum will give the greater angle; and ſubſtracting it, the leſs; by prop. IX. part I, II. Q. E. D.

SCHOL. I.

To ſolve this caſe by inſtruments, the preparation is the ſame as in the ſolution above, and the proportion is wrought in the following manner,

I. By the ſingle ſcale of tangents.

The diſtance from the ſum of the ſides (143) to their difference (23), on the ſcale of numbers, applied to the ſcale of tangents will reach from (25°.) half the ſum of the unknown angles, to (4°. 17′.) half their difference *.

II. By

* The tangents above 45° increaſe juſt in the ſame proportion as thoſe decreaſe which are the ſame number of degrees below it; v. g. the tangent of 50° is as much longer than that of 45°, as

II. By the double ſcale of tangents.

Take the ſum of the ſides (143) a lateral diſtance on the ſcale of lines and make this the tranſverſe diſtance of half the unknown angles (25°) on the ſcale of tangents; then take the difference of the ſides (23) a lateral diſtance on the ſcale of lines and apply it to the ſcale of tangents, the degree (4° 17′), where it becomes a tranſverſe diſtance, is the angle wanted.

N. B.

as that of 40° is ſhorter. Now the ſcale of tangents reaches but to 45°; from thence upwards they are reckoned backwards, and the ſame degree ſtands for 50 and 40, 60 and 30, &c. by this contrivance the ſcale is only ſo long as had been neceſſary to lay half down all the tangents. This however occaſions ſome difficulty in meaſuring intervals from a degree above 45° to one below it. Suppoſe in the given example the given things had been A B = 83, B C = 103 and the angle B 20°, the ſum of the given ſides had then been 186, and there difference 20; half the ſum of the unknown angles had been 80, and the half difference had been 31°. 23′. Now in this inſtance the firſt tangent 80° is above 45° and ought to have been as much to the left hand of 45°. as 80° (i. e. 10°) is to the right, and, had it it ſtood there, the compaſſes opened to the diſtance from 186 to 20 on the line of numbers, would have reached from 80°. to 31°. 23′. but if one foot be placed in 45°, (the higheſt point on the preſent ſcale,) the other will reach too far down by a ſpace equal to the diſtance from 80°. (i. e. 10°) to 45°. In this caſe, therefore, opening the compaſſes to the proper diſtance, place one foot at 45°, and let the other reach where it will in the lower tangents; then keep the lower foot ſteady and bring the upper from 45° to 80° (i. e. 10°); the opening of the compaſſes will now be leſſened by the ſpace from 80° to 45°, and conſequently if this extent be applied downwards from 45° it will reach the proper diſtance, 31° 23′.

N. B. If the ſum of the ſides be longer than the ſcale of lines, $\frac{1}{2}$, $\frac{1}{3}$ or any other part of the ſum and difference may be taken; or any other ſcale of equal parts may be uſed.

——If half the ſum of the unknown angles be more than 45°. and the half difference leſs, the ſides muſt be made a tranſverſe diſtance of the upper tangents, then the tranſverſe diſtance of 45° on the upper muſt be made the tranſverſe diſtance of 45° on the lower tangents, and the half difference be taken from thence.

SCHOL. II.

If the given angle be a right one between two ſides which are likewiſe given, a much eaſier way of finding the reſt will be laid down prop. XII. ſch. IV. caſe 6.

PROP. XII.

In any plane triangle, if all the ſides be given, to find the angles.

SOL.

Draw a line (which call a cathetus) from the largeſt angle perpendicular to the oppoſite ſide; that ſide will then be divided into two ſegments, and the triangle into two right angled triangles: then find the difference of the ſegments, thus,

As the baſe, or ſum of the ſegments,
is to the ſum of the two other ſides;
ſo is the difference of thoſe ſides
to the difference of the ſegments of the baſe.

Add

Add half the difference of the ſegments to half the baſe for the larger ſegment; ſubſtract d°. from d°. for the leſs, comp. prop. IX. part I, II.

Two ſides and an angle oppoſite to one of them in each triangle being now known, the others may be found by prop. X. caſe 1.

EXAMPLE.

In the triangle A B C (fig. 16.) A B = 75, A C = 37, and B C = 87; what are the angles? Draw A D perpendicular to B C.

A B 75	A B 75		
+A C 37	−A C 37		
ſum 112	diff. 38,		
Then ſay, as B C 87			1.93952
is to A B + A C, 112			2.04922
ſo is A B − A C, 38			1.57978
			3.62900
to B D − D C (difference of ſeg.) 49			1.68948
2) B C = 87	2) B D − D C = 49		
43½ the baſe	24, ½ difference of ſegments of baſe. 43 + 24 i. e. 67 = the greater ſegment B D. 43 − 24 i. e. 19 = the leſs ſegment C D. Then ſay		
As A C. 37			1.56820
is to D C 19			1.27875
ſo is the ſine of A D C = 90°			10.
			11.27875
to the ſine of C A D = 30°. 54′			9.71055

Since A D C = 90°, ſubſtract 30°. 54′. from 90°. it leaves A C B = 59°. 6′.

Then ſay again

As A B 75	—— ——	1.87506
is to B D 67	—— ——	1.82607
ſo is the ſine of A D B 90°		10.
to the ſine of B A D 63°. 18′,		11.82607
		9.95101

Since B D A = 90°, ſubſtract 63°. 18′. from 90°. it leaves A B C = 26°. 42′. Then ſubſtract A C B 59°. 6′ + A B C 26°. 40′, i. e. 85°. 48′, from 180°, 94°, 12′. will be left for B A C; and thus all the angles are found.

Demonstration.

Lemma: all the parts of the ſolution being ſelf-evident, or ſupported by former propoſitions, excepting the proportion; there is no need to prove any other part of it.

Construction.

On A with the radius A C deſcribe a circle; produce B A to E and draw A F: then

1. B C is the ſum of the ſegments of the baſe.
2. Since A E = A C *(radii)* B E = ſum of the two other ſides.
3. Since A G = A C *(radii)* B G = difference of the ſides.

4. Since DC = DF (Eucl. III. 3.) BF = difference of the ſegments of baſe.
Eucl. III. 37 cor. 5. BC × BF = BE × BG.
5, Eucl. VI. 16. 6. BC : BE :: BG : BF.
6, 1, 2, 3, 4. 7. Sum of ſegments : ſum of other ſides :: difference of ſides : difference of the ſegments Q. E.D.

SCHOL. I.

If two of the given ſides are equal, the cathetus will biſect the baſe, and the angles in one triangle will be equal to the correſpondent angles in the other.

SCHOL. II.

This caſe may be ſolved by inſtruments; thus,

I. By the ſingle ſcales; the diſtance from the baſe, (BC = 87) to the ſum of the other ſides (112) taken on the ſcale of numbers, will, on the ſame ſcale, reach from (38) the difference of thoſe ſides to (49) the difference of the ſegments of the baſe. In all the ſubſequent proportions, the diſtance from one ſide to another on the ſcale of numbers, will reach from 90° to the angle required, on the ſcale of ſines; thus, from 37 to 19 of numbers, will reach from 90° to 30° 54′ of ſines; and from 75 to 67 of the numbers, will reach from 90°, to 63° 18′. of the ſines.

II. Any one angle may be found by the double ſcales; thus, (v. g. C) take the ſide oppoſite to the angle ſought (AB = 75) a lateral diſtance on the

the ſcale of lines, open the ſector till this extent will reach on the ſame ſcale from (87 = B C) one of the remaining ſides to (37 = A C) the other; than take the tranſverſe diſtance of 30°, this being applied laterally to the ſcale of lines will reach to a degree which is half the angle ſought (29° 33′ = half of C).

The other angles may be found in the ſame manner, or by either method laid down in the ſcholia to prop. X, XI.

SCHOL. III.

The following examples may afford an opportunity of rendering theſe directions familiar by practice.

Example I. (fig. 14.) *.

Given B C 195
C 90°
B 47° 55′
Anſ. A 42° 5′
A B 291
A C 216.

Example II.

Given B C 117
C 134° 46′
B 22° 37′
Anſ. A 22° 37′
A C 117
A B 216.

Example III.

Given B C 408
C 22° 37′
B 58° 7′
Anſ. A 99° 16′
A B 159
A C 351.

Example

* The ſame diagram is uſed in all theſe examples to prevent the multiplying of figures, but the learner may draw triangles according to the data in each example, and mark the given ſides and angles before he begins his calculation.

Example IV.

Given B C 195
A B 291
C 90°
Anſ. A 42° 5′
B 47° 55′
A C 216.

Example V.

Given B C 216
A B 117
C 22° 37′
Anſ. A 134° 46′ *
B 22° 37′
A C 117.

Example VI.

Given B C 408
A B 159
C 22° 37′
Anſ. A 99° 19′ †
B 58° 4′
A C 350.9

Example VII.

Given B C 408
A B 159
B 58° 7′
Anſ. A 99° 16′
C 22° 37′
A C 351.

Example VIII.

Given B C 117
A C 117
C 134° 46′
Anſ. B 22° 37′
A 22° 37′
A B 216.

Example IX.

Given B C 195
A C 216
C 90°
Anſ. B 47° 55′
A 42° 5′
A B 291.

Example X.

Given A C 195
A B 216
B C 291
Anſ. B 42° 5′
C 47° 55′
A 90°.

* The ſupplement of 45° 14′ becauſe A is obtuſe.
† The ſupplement of 80° 41′ becauſe A is obtuſe.

Example XI.		Example XII.	
Given	B C 408	Given	A B 117
	A C 351		A C 117
	A B 159		B C 216
Anſ.	A 99° 16′	*Anſ.*	B 22° 37′
	B 58° 7′		C 22° 37′
	C 22° 37′		A 134° 46′

SCHOL. IV.

There are many other ways of ſolving triangles; but as the three laſt propoſitions ſhew how to ſolve them in all caſes where it can be done, it is not neceſſary to lay down and demonſtrate a greater variety of proceſſes: however, as there is an eaſier method of ſolving right-angled triangles, it may not be improper to lay down the principles of it.

In a right-angled triangle, as A B C (fig 17.) the hypothenuſe A B, or either of the legs, B C or A C, may be made radius.

Obſ. 1. If A B be radius, A C will be the ſine of B *, and then A B : A C :: the radius of the tables : the ſine of B.

For, ſuppoſe F B to be the radius on which the tables are formed, F D will be the ſine of B: but, ſince the triangles A B C and F B D are ſimilar, A B : A C :: F B : F D.

Obſ. 2. If one of the legs, as B C, be radius, A C will be the tangent of B †; and B C : C A :: radius of the tables : the tangent of B.

For,

* And B C the ſine of A.

† And if A C be radius, B C will be the tangent of A.

For, suppose B E to be the radius on which the tables are formed, E G will be the tangent of B; and because the triangles B C A and B E G are similar, B C : C A :: B E : E G.

The business of right-angled triangles is commonly proposed under 7 cases, the solution of all which depends on these two observations.

Case 1. Hyp. and angles, A B and B, given; required one leg, A C.

Ans. Rad. : sine of B :: A B · A C.

Case 2. Hyp and one leg, A B and A C, given; required the angles.

Ans. A B : A C :: radius : sine of B; and $90° - B = A$.

Case 3. Hyp. and one leg, A B and A C, given; required the other leg, B C.

Ans. Find the angle A by *case* 2, and then B C by *case* 1.

Case 4. The angles and one leg, A C, given; required, the hyp. A B.

Ans. Sine of B : radius :: C A : A B.

Case 5. The angles and one leg A C given; required the other leg, B C.

Ans. Radius : tangent of A :: A C : B C.

Case 6. The two legs, A C and B C, given; required, the angles.

Ans. B C : C A :: radius : tangent of B; and $90° - B = A$ *.

Case 7. The two legs, B C and A C, given; required, the hyp. A B.

* This is the case solved in prop. XI. In regard to right-angled triangles this is an easier method, but the other will answer in any kind of triangle.

Anſ. Find the angles by *caſe* 6, and then A B by *caſe* 4.

Theſe ſeveral caſes may eaſily be tried, ſuppoſing A B = 35, B C = 28 and A C = 21; A = 53° 8′, B = 36° 52′ and C = 90°.

SCHOL. V.

From what has been ſaid above it will be eaſy to ſolve theſe caſes by inſtruments; but it is to be remembered 1) that when radius is compared with a ſine, the ſine of 90° muſt be taken inſtead of radius: 2) when radius is compared with a tangent, the tangent of 45° muſt be taken inſtead of radius: 3) where the firſt proportional is leſs than the ſecond, the third will be leſs than the fourth; and therefore, in working a proportion in this caſe by the double ſcale, the 45th degree of the upper tangents muſt be ſubſtituted for radius; and *vice verſa*, the lower.

AN EASY

INTRODUCTION

TO PLANE

TRIGONOMETRY.

PART V.

A ſpecimen of the uſe of trigonometry in the application of it to the meaſuring of heights and diſtances, and to land-ſurveying.

CHAP. I.

Of meaſuring HEIGHTS.

PROP. XIII.

TO meaſure the angle G A H, (fig. 18.) by the quadrant, A H being a horizontal line *.

SOL.

The center being placed in A, let the obſerver at B hold the quadrant ſo as that he can ſee the

 object

* The quadrant is uſed to meaſure angles in planes perpendicular to the horizon, for all other angles the theodolite.

object G thro' its ſights; the angle C A D is equal to that required; and C A D is known by obſerving what degree and minute in the ſeries that begins at C is cut by the plumb line A D.

DEMONSTRATION.

	1. By the conſtruction of the quadrant G A C is a right angle.
	2. Since A H is horizontal, H A D muſt be a right angle.
1, 2.	3. G A C = H A D.
3.	4. Taking H A C from each, there will remain G A H = C A D.

SCHOL.

The remaining angle B A D is equal to Z A G, the angle made by the given line A G with a line paſſing thro' the obſerver's zenith.

PROP. XIV.

To meaſure the angle A B C (fig. 19.) with the theodolite.

SOL. AND DEM.

Place the center of the theodolite at B, ſo that A may be ſeen thro' the fixed ſights; there fix it, and turn the moveable ruler till C is ſeen thro' its ſights: It is ſelf evident that the degrees from D to E, marked on the circular limb from D to E, will be the meaſure of A B C.

PROP.

PROP. XV.

To meaſure the height of an acceſſible tower; v. g. A B, (fig. 20.) C B being a horizontal line.

SOL.

Meaſure C B, and take C by prop. XIII. A B C is a right angle; find C A B by taking C + A B C from 180°: all the angles and one ſide being now known, the other ſides, and particularly A B, may be found by prop. X.

SCHOL. I.

No calculation will be neceſſary if a perſon can go backward or forward till the angle at C is of 45°. for then the triangle is an iſoſceles, and height = diſtance.

SCHOL. II.

If the tower ends in a point, as D (fig. ib.) the diſtance from the obſerver is to be accounted to a perpendicular let fall from D, i. e. C G not C B: half the thickneſs of the tower being added.

SCHOL. III.

To find the true length of A B, in this and the following caſes, the height of the obſerver's eye muſt be added to the ſide of the triangle which is oppoſite to him, as found by calculation.

PROP. XVI.

To meaſure an inacceſſible height from a horizontal plane: v. g. A B, (fig. 21.) D C being a horizontal plane, and C being in the line from D to B.

SOL.

Take A C B and thence calculate A C D (i. e. 180°—A C B); meaſure D C and take A D C; having found two angles of the triangle A D C the third is had of courſe; and now, all the angles and one ſide (D C) being known, A C may be found by prop. X.

In the triangle A C B, the ſide A C is now known and the angles A C B and A B C *(a right angle)*; therefore A B may be found by prop. X.

PROP. XVII.

To meaſure an inacceſſible height, when a horizontal plane cannot be found, on which the obſerver may approach or retire from it: v. g. A B (fig. 22).

SOL.

Take two ſtations C and D, the farther aſunder the better; meaſure the angles A D C and A C D with the theodolite; calculate D A C, and meaſure D C: all the angles and one ſide of the triangle A D C being known, A C may be found by prop. X.

A B C is a right angle, take A C B with the quadrant, and calculate the third; all the angles of

of the triangle A C B and one side (A C) being known, A B may be found by prop. X.

PROP. XVIII.

A tower being placed on a hill, to find the height of it, separate from that of the hill; as A E (fig. 21, 22); or a spire, D E (fig. 20.) separate from its base.

SOL.

Having by prop. XV, XVI, or XVII, found A C, and all the angles of the triangle A C B, take E C B; subtracting E C B from A C B leaves A C E, and A C E + C A E from 180° leaves A E C: all the angles of the triangle A C E and the side A C being now known, A E may be found by prop. X.

CHAP. II.

Of measuring DISTANCES.

PROP. XIX.

From a given height, as A (fig. 20.) to find the distance of C.

SOL.

A B C is a right angle, take B A C with the quadrant, and calculate A C B; all the angles of the triangle A B C being now known and one side

ſide (A B); A C the direct, or B C the horizontal, diſtance of C may be found by prop. X.

PROP. XX.

To meaſure the diſtance of two places (A and B fig. 23.) of which one (B) is acceſſible.

SOL.

Take C at a convenient diſtance from B and A; meaſure B C; find, by the theodolite, the angles B, and C, and (by ſubſtracting their ſum from 180°) A. All the angles and one ſide being now known, A B may be found by prop. X.

PROP. XXI.

To meaſure the diſtance of two places (A and B fig. 24.) neither of which are acceſſible.

SOL.

Set marks at C and D (the farther aſunder, and the nearer to a paralleliſm with A and B, the better): at C take, with the theodolite, A C D, and B C D; and at D, take A D C, B D C and B D A: meaſure C D.

Then in the triangle A C D, all the angles and one ſide (C D) being known, A D may be found by prop. X.

In the triangle B C D, all the angles and one ſide (C D) being known, the ſide B D may be found by prop. X.

And in the triangle A D B, the ſides A D and D B being known and the angle (A D B) between

tween them, the ſide A B may be found by prop. XI.

PROP. XXII.

To make a draught of remarkable objects in a town or country, ſhips in a harbour, &c.

SOL.

Suppoſe the objects to be A, B, C, D and E (fig. 25).

Take two ſtations F and G at a convenient diſtance; at F take, with the theodolite, the angles A F G, B F G, C F G, D F G and E F G; meaſure the diſtance (ſuppoſe 500 yards) from F to G, and at G take the angles E G F, D G F, C G F, B G F and A G F.

Draw a line of 500 equal parts taken from a proper ſcale, and at the ends of it lay down angles equal to thoſe found by obſervation. The diſtance from the point which anſwers to F, to that in which the lines from the two ends interſect, being applied to the ſame ſcale, will ſhew the diſtance of the objects from F; and their diſtance from G and from each other may be found in the ſame manner.

Or the diſtance may be calculated by prop. X. as two angles of each trangle A F G and A G F, &c. being given, the 3d is eaſily found, and the ſide oppoſite to it, F G, is meaſured.

PROP. XXIII.

To find the horizontal line of a hill; v. g. A G D (fig. 26).

CON-

CONSTRUCTION.

Drive down a ſtake at B and C; let A D and B C be horizontal; B E and C F perpendicular to A D.

SOL.

Meaſure A B and with the quadrant take the angles B A E and A B E, and by prop. X. find A E: in the ſame manner find F D. Then A E + B C + F D will be the horizontal line of the hill.

PROP. XXIV.

To meaſure the diameter of the earth.

SOL.

Take exactly the height of a mountain near the ſea; from the top of it, as A, (fig. 27.) with a good quadrant (divided into ſeconds, and a ſmall teleſcope inſtead of ſights,) take the angle D A E, comprehended between A D, a tangent to the earth at the point D, and the diameter A E; and let C B be drawn a tangent to the earth at C: then in the triangle A C B, C A B being found and A C B given, all the angles and one ſide (A C) are known, and the other ſides A B and B C may be found by prop. X. But B C and B D, being tangents from the ſame point B, are equal, by a corollary from Eucl. III. 36; therefore A B + B C are = A B + B D, i. e. A D: thus the length of A D is found. But, by Eucl. III. 36. AD^2 is $\doteq AC \times AE$; and therefore (by Eucl. VI. 17.) A C : A D :: A D : A E. Now, ſince A C and

A D

A D are known, A E may be found by the rule of three; and, ſubſtracting from it A C (the height of the mountain), there will remain C E, the diameter of the earth.

SCHOL.

A more certain method of finding the diameter of the earth is by meaſuring a degree on its ſurface.

A degree of the meridian at *lat.* 49°. 21′. was found to be 57183 french toiſes; or, (a toiſe=6 feet) 343098 french feet; or (a french foot containg 12.788 inches, engliſh) 365628 engliſh feet; or, 121876 engliſh yards; or, nearly 69.25 engliſh miles; and then a whole circle, 360°, will be 24930 miles: and ſince 355:113 as periphery to diameter, 7935.47 miles will be the diameter of the earth.

N. B. This ſuppoſes the earth a perfect ſphere; but a degree in lapland, *lat.* 66°. 20′, being found to be 344628 paris feet, if the meaſures are juſt, the polar diameter is to the equatorial, as 177 is to 178; or the earth is 22 miles higher at the equator than the poles.

CHAP.

CHAP. III.

Of LAND-SURVEYING.

PROP. XXV.

To deſcribe the principal inſtruments employed in ſurveying land, and the manner of uſing them.

SOL.

1. A *Gunter's chain.* It is 22 yards long, and conſequently 80 chains make a mile. It conſiſts of 100 links each 7.92 inches long. An acre is one chain in breadth and ten in length, or 100 × 100 × 10, i. e. 100000 ſquare links *. Hence when any piece of ground is meaſured by the chain, reduce it into links and cut off 5 places to the right hand, the reſt are acres; multiply the figures cut off by 4, and cut off 5 places as before, the reſt are roods; multiply the figures cut off by 40, and again cut off 5 places, the reſt will be poles. If the figures laſt cut off are 50000, they make half a pole; but they may generally be neglected.

Each chain conſiſting of 100 links, any number of chains and links may be reduced to links without multiplication; thus, 48 chains and 73 links are 4873 links; but if the figure for links be an

* A pole or perch is $5\frac{1}{2}$ yards: the gunter's chain is therefore 4 poles long. By the ſtatute an acre contains 160 ſquare poles, i. e. 4 × 4 × 10. 40 Poles make a rood, and 4 roods an acre.

an unit, a cypher muſt be put before it; thus, 37 chains and 7 links are 3707 links.

Example. Suppoſe a parallelogram to be 48 ch. 73 links × 54 ch. 23 links.

Q. What is its area?

		4873
		5423
		14619
		9746
		19492
		24365
Anſ. 264 acres, 1 rood and 2 poles.	acres	264|26279
		4
	rood	1|05116
		40
	poles	2|04640

2. Three or four *flags* on poles about 6 feet long; and 10 arrows, each about 2 feet.

The flags are ſet up as marks at the corners, &c. of a field. Two perſons uſe the chain: the aſſiſtant, taking the arrows, carries off one end of it from the place where the meaſure is to begin, the ſurveyor keeping his eye on the flag, directs him in the line to it, and informs him when the whole chain is drawn out: then the aſſiſtant puts down the firſt arrow, and proceeds with his end of the chain, under the direction of the ſurveyor as before, till the other end of it comes to the firſt arrow; he then puts down a ſecond arrow, and the ſurveyor takes up the firſt. Thus they proceed till at the 11, 21, 31, &c. chain, all the ar-

rows,

rows being in the hand of the ſurveyor, they muſt be carried to the aſſiſtant. Thus an error can hardly be made, ſince it cannot be of leſs than 10 chains at once.

3. An *off ſet ſtaff*, 10 links in length, and divided into 10 equal parts, for meaſuring the diſtance of objects from the ſtation line, and to adjuſt the chain if any of the links are ſtretched.

4. A *croſs-ſtaff* to find what points are perpendicular to a hedge, line, &c. It conſiſts of two parts; a round piece of board 1 $\frac{1}{2}$ inch in thickneſs, and 5 inches diameter; on the upper ſide are two channels about an inch deep, made by a fine ſaw, exactly at right angles to each other, and in the center of the under ſide is a hole to receive the end of the other part, which is a ſtaff 4 or 5 feet long, to fix in the ground and ſupport the top in a horizontal poſition.

5. A *theodolite* to meaſure angles. This cannot eaſily be deſcribed, and it is the leſs neceſſary, as no deſcription will be of much uſe without the inſtrument itſelf, and the bare inſpection of it will ſhew its uſe.

N. B. Inſtead of the theodolite, ſurveyors now commonly uſe a *plane table and index*. The table is a level board about the ſize of a ſheet of paper, mounted, as the theodolite, on three legs, and ſurrounded with a moveable frame to keep a ſheet of paper tight upon it. The ſides of the frame next the paper are divided into equal parts, and the outer edges into degrees and minutes. A compaſs is annexed to one ſide of the table. The index is a braſs ruler about 2 feet long; about 3 inches of each end

are

are turned upwards at right angles to the middle, in which are the ſights; one edge of the ruler is exactly parallel to the ſights, and commonly in a line with them; and on the upper flat ſide there are ſcales of cords, equal parts, &c. The chief inconvenience of this inſtrument is that the leaſt rain, or even a damp air, will ſpoil the paper.

6. A *field-book* in which to make rough draughts, enter the meaſure of ſides and angles, note remarkable objects, &c. From theſe memorandums an accurate map is to be made at home.

7. A *caſe of inſtruments*; particularly a protractor, ſector, or ſcale of chords, to lay down angles, and a ſcale of equal parts to draw the ſides of a figure proportionable to the ſides of a field, &c. One ſide of the ivory ſcale is particularly deſigned for this. The whole length is divided into half inches, each anſwering to a chain, and one half inch, by 10 lines drawn a-ſlant 10 parallel lines, is divided into 100 equal parts, anſwering to links: this is called the diagonal ſcale, as each croſs line is the diagonal of an imaginary parallelogram, which has the breadth of the ſcale for its length, and $\frac{1}{10}$ of $\frac{1}{2}$ an inch for its breadth. There is another ſcale, where $\frac{1}{4}$ of an inch anſwers to a chain for drawing ſmaller plans.

PROP. XXVI.

To delineate a field from one ſtation by the theodolite.

SOL.

Make a rough draught of it in the field-book, and chooſing a ſpot in the field from whence all

the corners of it may be ſeen, take a point within the draught to repreſent it, as A (fig. 28; place letters at the angles of the figure which anſwer to the corners of the field, and draw lines from A to each.

Direct the fixed ſights of the theodolite to the corner that correſponds to B, and the moveable ſights to C; meaſure the diſtance to each corner, enter the diſtance in the field book, and the angle as marked on the circular limb of the theodolite. Thus take and regiſter the diſtance to the ſeveral corners from A, and the angles made at that point by lines drawn to the ſeveral corners. Then the field-work is done.

N. B. If the ſum of the angles be more or leſs than 360°, there is a miſtake.

Suppoſe the regiſter to be.

A B	4 ch. 92 links,	D A E	105°
B A C	61°	A E	4 ch. 73 links,
A C	5 — 44	E A F	65°
C A D	75°	A F	5 — 64
A D	4 — 94	F A B	54°

At home, take from the ſcale 4 ch. 92 links, and draw A B (fig. 29.) of that length; then take 5 ch. 44 links for the length of A C, and make B A C=61°: thus lay down all the lines and angles as entered in the field book, and thro' the extremities of the lines draw B C, C D, D E, &c. the figure will be the out-line of the field.

SCHOL. I.

To perform this by the plane table, ſuppoſe A B C D E (fig. 30.) to be the field, and F G the

the inſtrument fixed in a place whence all the corners may be ſeen, let a proper point of the paper, as *x*, be choſen; apply the index to that point, and directing the ſights to A, draw a faint line along its ſide; meaſure the diſtance to A, and, taking the number of chains and links from the ſcale, make *x m* of that length. The table remaining fixed, direct the index, ſtill applied to the point *x*, to B; draw a faint line, meaſure the diſtance to B, and by the ſcale make *x n* proportionable to it. Thus proceed as to all the corners, and draw the ſtrong lines *m n*, *n o*, *o p*, &c. it is ſelf-evident that the ſides of the figure will be parallel to the ſides of the field, and therefore a delineation of it.

SCHOL. II.

If there be no place from whence all the corners of the field can be ſeen, it may be divided by an imaginary line drawn between any two convenient corners, and meaſured as if it were two fields.

PROP. XXVII.

To delineate a field from a ſurvey taken at two ſtations, from each of which all the corners may be ſeen.

SOL.

Call one ſtation A, the other B; make a rough draught of the field in the field-book, diſtinguiſhing the corners by letters; fix on two points in the draught which may repreſent A and B; draw a line from A to B, and from each of them to

each corner (ſuppoſe fig. 31. to be the rough draught).

Fix the inſtrument at the ſtation A with the immoveable ſights directed to C; take and regiſter the ſeveral angles C A D, D A E, E A F, F A G, G A C and B A C, and the diſtance from A to B.

Remove the inſtrument to B; and fixing its immoveable ſights to C, firſt meaſure and regiſter C B A, and then C B D, D B E, E B F, F B G and G B C. The field-work is done.

Suppoſe the regiſter to be

At A				At B			
C A D	72°	F A G	38°	C B A	101°	E B F	59°
D A E	115°	G A C	73°	A B 4 chains	C B D.	63°	F B G 47° 30′
E A F	62°	B A C	59°		D B E	77°	G B C 113° 30′

At home, fix on A and B at 4 chains on the ſcale aſunder, at A draw a line of ſufficient length to repreſent A C, making with A B an angle of 59°; in this manner lay down all the angles at A, as regiſtered in the field-book. The corners muſt fall in the lines which proceed from A.

From B draw a line to repreſent B C making with B A an angle of 101°, and lay down all the other angles at B, as regiſtered in the field book, the corners muſt likewiſe fall in the lines which proceed from B, therefore they can only fall in the points where the lines from A and B interſect, as at *c*, *d*, *e*, *f*, *g*, (fig. 32.) lines drawn thro' theſe points will repreſent the field, and the ſides of the figure applied to the ſcale will meaſure the ſides of the field.

SCHOL.

SCHOL. I.

The diſtance from A to B ſhould be conſiderable in proportion to the field; one of them ſhould not be in the line from the other to any corner of the field; and they ſhould be ſo taken that none of the angles formed by lines from one with thoſe from the other, may be very obtuſe or accute: on this account this method is rather curious than uſeful.

SCHOL. II.

To do the ſame by the plane table; ſuppoſe the given field to be C D E F (fig. 33.) fix the table as at G H, and taking a proper point in the paper to repreſent that ſtation, as A, apply the index to it, and directing the ſights ſucceſſively to all the corners of the field, draw lines towards each. While the table remains, fix on a ſecond ſtation, meaſure the diſtance, apply the index to A, and direct its ſights to the ſecond ſtation. Make A B by the ſcale proportionable to the diſtance of the ſtations. Remove to the ſecond ſtation; let the line B A point to the firſt ſtation, and there fix the table. Apply the index to B, direct it ſucceſſively to all the corners of the field and draw faint lines; lines drawn thro' the points where theſe and the former interſect will repreſent the field.

PROP. XXVIII.

To delineate a field by obſervations made with the theodolite in going round it.

SOL.

Let A B G H I C D E F M L K, (fig. 34.) be the field: place flags at A, B and C, and meaſure A B; at B direct the fixed ſights to A and the moveable to C, thus take the angle A B C. In this manner meaſure and regiſter all the ſides and angles, carefully diſtinguiſhing the external angles, if there be any (as D).

At G in the ſide B C a corner runs out; however, as it is a ſmall one, the ſtation-line need not be diverted, but may run on to C: and thus at M in the ſide F A a piece is taken out of the field: with the off-ſet-ſtaff meaſure the baſes and perpendiculars of G H I and M L K, enter them in the field book, the ſhape as approaching neareſt to a parallelogram, a triangle or ſegment of a circle, and whether they are to be added or deducted.

N.B. If a map is to be made, the diſtance from ſome corner (as B G and F M) muſt be taken.

Suppoſe the regiſter to be

A B	7 ch. 96 links,	D E	3 ch. 0 links,
A B C	—— 79° 30′	D E F	—— 96° 30′
B C	8 — 64	E F	4 — 81
B C D	—— 57°	E F A	—— 112°
C D	2 — 8	F A	6 — 48
C D E	(external) 88°	F A B	—— 103°

To be added

G I	1 ch. 50 links	} tri-
Per. hn,	0 — 71	} angle.
N.B. B G	4 — 8.	

To be deducted.

K M	1 ch. 16 links	} ſegt. of
Perp. lo	0 — 37	} circle.
N.B. F M	2 — 41	

At

At home, deſcribe a figure from the regiſter in the field-book.

SCHOL. I.

All the internal angles are equal to twice ſo many right angles deducting four; but for an external angle its complement to 360° muſt be taken. By this rule the truth of the work may be tried.

SCHOL. II.

This may be done hy the plane table in ſo eaſy a manner, that it ſeems unneceſſary to deſcribe it, after what has been ſaid in the ſch. to the two laſt propoſitions, concerning the manner of uſing that inſtrument.

SCHOL. III.

If the bounds of a field, ſuppoſe A B C (fig. 35.) cannot be meaſured, it muſt be ſurveyed on the inſide, or outſide: ſuppoſe the former; fix flags at D and E parallel to A B; by the croſs-ſtaff find G, the point where a line from D perpendicular to A B will fall, meaſure G A with the off-ſet-ſtaff; in the ſame manner find H and meaſure H B: add A G and H B to D E for the ſide A B; a flag being placed at F in the ſame ſituation in reſpect to C as the others are to A and B, D E F may be taken for A B C, and all the other ſides and angles meaſured in the ſame manner.

PROP. XXIX.

To meaſure the area of a field from a delineation of it.

SOL.

Let A B C D E F G H I K L, (fig. 36.) be the delineation of a field, made by prop. XXVI, XXVII, XXVIII, or the ſcholia.

Divide it into the triangles A B C, A L C, L C G, and D E F, and the portion of a circle H I K.

Let fall the perpendiculars B *o*, A *p*, G *q*, E *n* and *km*. To prevent miſtakes, write on theſe, and the baſe of each, the length, as found by the ſcale. Then find the area of each portion; add the areas of A B C, A L C, L C G and D E F into one ſum, ſubſtract from thence H I K, and find the acres, roods, &c. in what remains by prop. XXV. gr. 1.

N. B. A perpendicular let fall from an angle to the baſe gives the height of a triangle; and from the higheſt point of an arc to the chord, of a ſegment of a circle. The baſe of a triangle multiplied into $\frac{1}{2}$ the height gives its area (by Eucl. I. 41,); to find the area of a ſegment of a circle, multiply the cord into $\frac{2}{3}$ of its height; which, tho' not perfectly exact, is near enough for practice.

The operation will ſtand thus,

A C = 871	L C ——— 845
× $\frac{1}{2}$ *o* B 163.5	* × $\frac{1}{2}$ A *p* + $\frac{1}{2}$ G *q* = 376.5
4355	4225
2613	5070
5226	5915
871	2535
A B C = 142408.5 links.	A L C + L C G = 318142.5 links.

* As the triangles A L C and L C G have the ſame baſe, their perpendiculars are here added together, and the baſe multiplied by half their ſum at once.

DF =	128	ABC =	142408.5
$\times \frac{1}{2}$ En =	37	ALC + LCG =	318142.5
	896	DEF =	4736
	384		465287
DEF =	4736 links.	− HIK	1240
KH =	94	acres	464047
$\times \frac{2}{3}$ Im =	13.2		4
	188	roods	2156088
	282		40
	94	poles	2247520
HIK =	1240.8 links.		

PROP. XXX.

To measure a regular field, A B C D E (fig. 37), without any delineation.

SOL.

Divide it into the triangles A B C, A E C, E C D by imaginary lines, measure these lines, and enter their length in the field-book; find by the theodolite, cross-staff or any proper instrument, the points *o*, *m*, and *n* to which the perpendiculars from the opposite angles will fall; measure these perpendiculars, and find the areas of the triangles, and of the whole field as directed in the last proposition.

SCHOL. I.

In all these propositions it is supposed that a field is bounded by streight hedges: If they are crooked, station-lines may be made on the inside, and off-setts taken (the more the better) to the nearest and most distant parts of the hedges, at right

right angles to the ſtation lines; add the off-ſets of any ſide into one ſum and divide it by the number of them, the quotient multiplied by the correſpondent ſtation-line will give a parallelogram, which muſt be added to the area within the ſtation-lines.

SCHOL. II.

Theſe few propoſitions may ſuffice to give a general notion of ſurveying. Many little articles may be learnt by books written on the ſubject, or will occur to a perſon in practice, which is after all the beſt inſtructor.

APPENDIX.

Shewing the uſe of the inſtruments to ſome purpoſes which do not fall directly under any of the heads treated of above.

PROP. XXXI.

To make a triangle whoſe area ſhall be equal to that of any given quadrilateral figure, A B D C (fig. 38).

SOL.

Produce C D, draw A D, and, parallel to it, thro' B draw B E: then draw A E; A C E will be = A B D C. Dem-

DEMONSTRATION.

Since A D is by the conſtruction parallel to B E, the triangle B A D will (by Eucl. I. 37.) be = E A D; and therefore B A D + A C D (i. e. A B D C) will be = E A D + A C D (i. e. A C E).

SCHOL.

If it were required to make the vertex in any part of any ſide (as F fig. 39) the operation and proof would be much the ſame.

Thus, produce C D each way; draw F D, and, parallel to it, B E; F C, and, parallel to it A H: then, drawing F H and F E, F H E will be = A B D C.

For, if F G be drawn, it is ſelf-evident that the triangle F H E = F H G + F G E, and A B D C = A F G C + F B D G; but, by the argument in the propoſition, F H G is = A F G C and F G E = F B D G; and therefore F H E is = A B D C.

PROP. XXXII.

To reduce a pentagon, A B C D E (fig. 40.) to a triangle of equal area.

SOL. AND DEM.

Draw A C, and parallel to it, B F; produce D C to F, and draw A F; then ſince the triangle B A C is = F A C (Eucl. I. 37.) A F D E is = A B C D E; thus the pentagon is reduced to a quadrilateral figure, which may be reduced to a triangle by the laſt propoſition.

SCHOL.

SCHOL.

Any other right-lined figure may be reduced to a triangle of equal area in the like manner.

PROP. XXXIII.

To make a ſpiral of a given breadth, as N O (fig. 41).

SOL.

Draw the lines E H and F G at right angles; from O, the point of interſection, take *o a*, *o b*, *o c* and *o d*, each = to $\frac{1}{4}$ of N O; with *a* as a center, and the radius *a* E, draw the arc E I; with *b* as a center, and the radius *b* I, draw the arc I K; with the center *c*, and the radius *c* K draw K L; and thus proceed at pleaſure.

PROP. XXXIV.

To deſcribe an ellipſis by the double ſcale of ſines, the diameters being given.

SOL.

Let the two diameters, A B and C D (fig. 42), croſs each other in their middles, at right angles, as at E.

Make A E ($\frac{1}{2}$ the tranſverſe diameter) the tranſverſe diſtance of 90°; apply the tranſverſe diſtance of 15° each way from E, as at 1, 1; and the tranſverſe diſtances of 30, 45, 60 and 75° in the ſame manner, at 2, 3, 4 and 5; thro' the points thus taken, draw lines parallel to the conjugate diameter C D.

Make

Make E D ($\frac{1}{2}$ the conjugate diameter) the tranſverſe diſtance of 90°; from 1 and 1 ſet off on each ſide the tranſverſe diſtance of (90—15) 75°; from 2 and 2, of (90—30) 60°; from 3 and 3, of (90—45) 45°; from 4 and 4, of (90—60) 30°; and from 5 and 5 of 15°.

A curve line drawn thro' theſe points will be the ellipſis required.

SCHOL.

To make it more exact, the lines parallel to the conjugate diameter may be drawn at only 10 or 5° aſunder, and the diſtances ſet off from 1 and 1 muſt be 90°—10 or 90°—5, &c.

PROP. XXXV.

To deſcribe an ellipſis by a ruler and compaſſes only, the diameters being given.

SOL.

Let the two diameters, A B and D E (fig. 43,) croſs each other in their middles, at right angles.

From either end, D or E, of the conjugate diameter ſet off D F and D *f*, each equal to C A, half the tranſverſe diameter; the points F and *f* are the foci of the ellipſis. Set off any number of diſtances, C G, C H, &c. (no matter whether they are equal among themſelves or not) from C towards F. Take A G in the compaſſes, and ſetting one foot in the foci F and *f*, with the other foot deſcribe the arcs *a b*, *a b*, from F; and *c d*, *c d* from *f*: then take the remaining part of the

axis

axis B G, in the compaſſes, with which, as a radius, deſcribe *i k*, *i k*, from the focus F; and *g h*, *g h*, from the focus *f*. Take A H in the compaſſes, and with this length as a radius, from F deſcribe the arcs *l m*, *l m*, and from *f*, the arcs *n o*, *no*; then take the remaining part of the axis, B H, in the compaſſes, and with that extent, from F deſcribe the arcs, *r s*, *r s*; and from *f* the arcs *p q*, *p q*. Proceed in the ſame manner with all the reſt of the diſtances, and deſcribe arcs interſecting each other. Laſtly, thro' all theſe interſections draw the elliptical curve by hand. Mr. *Ferguſon*.

PROP. XXXVI.

To divide the periphery of a circle into any given number of equal parts, whether odd or even, or to leave any given fraction of a part at laſt.

Let A B D E (fig. 44.) be a circle to be divided into 365 $\frac{1}{4}$ equal parts. Subſtract 5 $\frac{1}{4}$ of theſe parts, there will remain 360, which may eaſily be divided into four equal parts, each containing 90. Then ſay 365.25 : 360° :: 5.25 : 5.147, or 5° and very little more than $\frac{1}{10}$ of a degree. Make radius A C equal to 60° of the ſcale of cords on a ſector, and taking 5 $\frac{1}{10}$ from the chords in the compaſſes, ſet it off from A to B in the periphery. Then divide the remaining part B D E A into four equal parts, each of theſe into 3, each of theſe ſubdiviſions again into 3, and each of theſe laſt into 10, the larger ſegment will be divided into 360 equal parts, of which the ſmaller will contain 5 $\frac{1}{4}$.

There

N.B. There is no number (beyond 4) but ſo many may be ſubſtracted from it as will leave a number which may be divided into 4 equal parts; and the arc equal to the ſubſtraction may be found as above.

Thus let it be required to divide a circle into 29 $\frac{1}{2}$ equal parts for the days of the moon's age: ſubſtract 1 $\frac{1}{2}$ from 29 $\frac{1}{2}$, there remains 28 which may be divided in 4 parts, and each of theſe by 7: then 29 $\frac{1}{2}$: 360° :: 1 $\frac{1}{2}$: a fourth number to be ſet off, as from A to B. Mr. *Ferguſon.*

PROP. XXXVII.

To make a horizontal ſun dial by the ſcale of latitudes and hours on the ſector.

SOL.

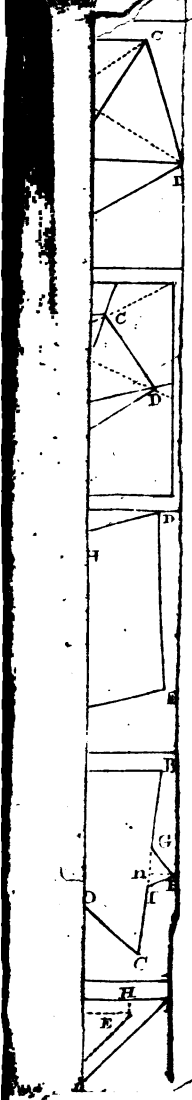
" Draw the double meridian *a b*, *c d*, (fig. 45.) " croſs it at right angles by the ſix o'clock line " *e f*: take the latiude of the place from the ſcale " of latitudes and ſet it from *c* to *e*, and from *a* to " *f*; then take the whole length of the line of " hours in the compaſſes, and placing one foot in " *e*, let the other fall where it will in the meridian " line *e d*, as at *d*: do the ſame from *f* to *b* and " draw *e d*, *f b*.

" This done, ſetting one foot of the compaſſes in " the beginning of the line of hours, and extend- " ing the other to each hour on the ſcale, lay off " theſe extents from *d* to *e* for the afternoon hours, " and from *b* to *f* for thoſe of the forenoon: this " will divide the lines *d e* and *b f* in the ſame man- " ner as the hour-ſcale is divided, at 1, 2, 3, 4, 5, " on which the quarters may alſo be laid down, if " required

" required. Then, laying a ruler on the point *c* " draw the firſt five hours in the afternoon, from " that point, thro' the dots at 1, 2, 3, 4, 5 on the " line *d e*, and continue the lines of IIII and V " thro' the center to the other ſide the dial, for the " like hours in the morning: this done, lay the " ruler on the point *a*, and draw the laſt five hours " in the forenoon thro' the dots 5, 4, 3, 2, 1, on " the line *f b*; continuing the hour lines of VII " and VIII thro' the center *a* to the other ſide of " the dial, for the like hours of the evening. Mr. " *Ferguſon's Philoſophical Lectures*, p. 347."

The upper edge of the ſtyle is to make an angle with the plane of the dial equal, in degrees and minutes, to the latitude of the place, the point of it muſt be at *a c.* and the ſpace between *a b* and *c d* muſt be equal to its thickneſs. As the hour-lines are leſs diſtant from each other about noon, than in any other part of the dial, it is beſt to have the center of the circle in which the hours are written, brought forwards towards the northern ſide of the dial: thus in the figure the center is at *o*, and had it been brought more forward, the diſtances had been more equal.

THE END.


C
C
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REIGLES
DES CINQ ORDRES
D'ARCHITECTVRE
DE VIGNOLLE
Reueües Augmentées et Redu
ittes de Grand en Petit
Par le Muet
Auec Priuilege du Roy
A Paris chez I. Mariette rue S.t
Iacques aux Colonnes d'Hercule
et a la Victoire.
ML

Auis au Lecteur.

Avant que passer outre jay jugé quil estoit a propos pour les moins intelligents en cette science donner a entendre que cest que module, qui nest autre chose quune mesure de la longueur du semy diamettre de la Colonne que voulez construire.

Exemple soit le diamettre de la Colonne A de 18. 16. ou 12. pouices ou quelque aues nombres son semy diamettre sera de 9. 8. ou 6. lequel semy diamettre sapellera module qui sera party en 12. parties esgalles, desquelles on vsera pour proportionner touttes les parties de ladite colonne.

Or il est a notter que cette partition du module en 12. parties esgalles ne se pratique quaes ordres Toscan, et Dorique, et que pour les autres ordres il se diuise en 18. parties esgalles.

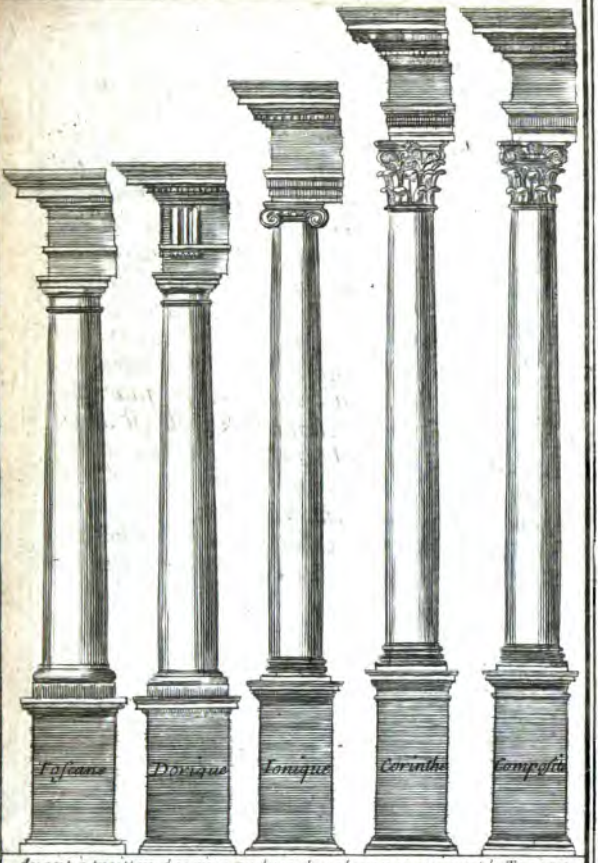
M. sinifie Modules P. sinifie partie des Modules.

REGLES
DES CINQ ORDRES
D'ARCHITECTVRE
DE VIGNOLLE

Reueuee Augmentées et Reduites
de Grand en petit par le Muet
1591 — 1669

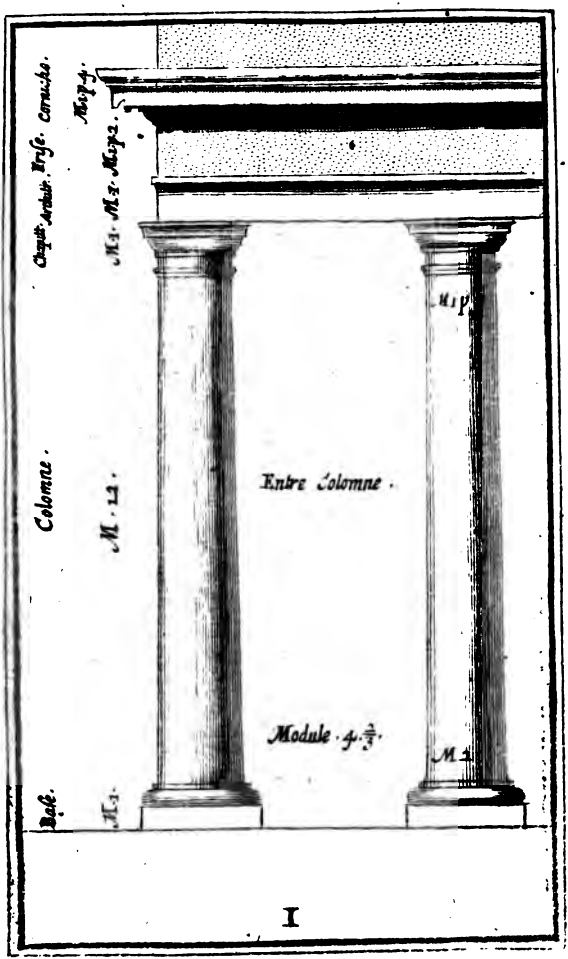
A PARIS
Chez I. Mariette rue S.t Iacques
aux Colonnes d'Hercule
et a la Victoire.

Auec Priuilege du Roy



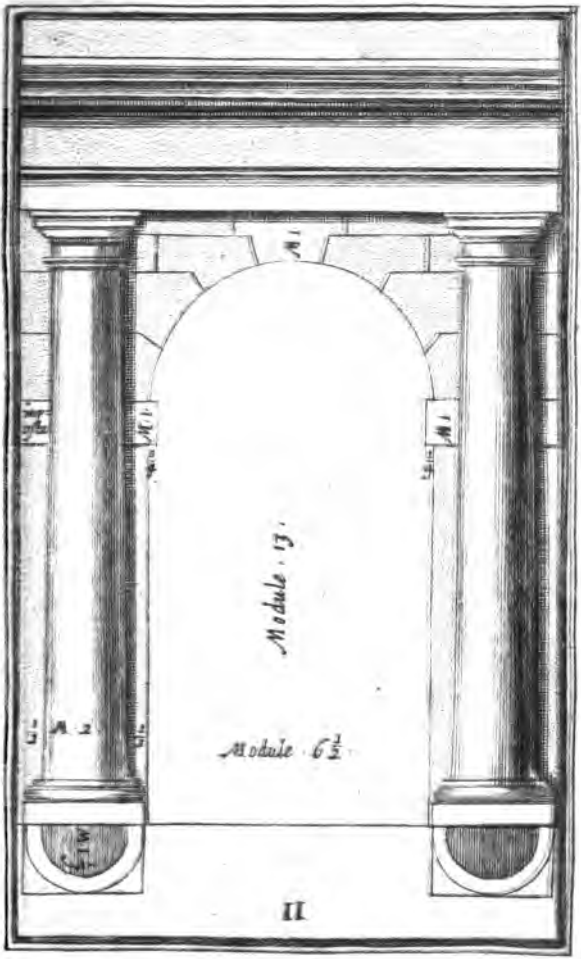
Ayant a traitter des cinq ordres de colomnes qui sont le Toscan le Dorique Ionique, le Corinthien, et le Composé; il ma semblé à propos de representer icy les figures de toutes les especes dont ie parleray, quoy que leur mesures particulieres ny soient pas marquées parce ce que ie les mets seulement comme pour une regle generalle qui se declarera dans la suitte de chacun des cinq ordres en particu

N'Ayant trouué entre les Antiquitez de Rome aucun ornement Toſcan du quel j'aye pû former vne regle comme j'ay bien fait des autres quatre Ordres Dorique, Ionique, Corinthien, & Compoſé je me ſuis ſerui de lautorité de Vitruue liure 4. chapit. 7. Ou il dit que la Colomne Toſcane doit auoir la hauteur de ſept de ſes propres groſſeurs auec la baſe & le chapiteau. Au reſte de l'ornement aſauoir l'architraue, la friſe, & la liſte ou Corniche il me ſemble choſe conuenable qu'on y garde la reigle que j'ay trouuée es autres ordres c'eſt aſauoir que l'Architraue, la friſe & la corniche ſoyent le quart de la hauteur de la Colomne, laquelle eſt de 14. Modules, auec la baſe et le chapiteau comme l'on voit ici noté par nombres. et ainſi l'Architraue, friſe et corniche tiendront 3. Modules $\frac{1}{2}$ qui eſt le quart de 14. ſes membres particuliers et ſeront marquez en detail chacun en leur lieu.

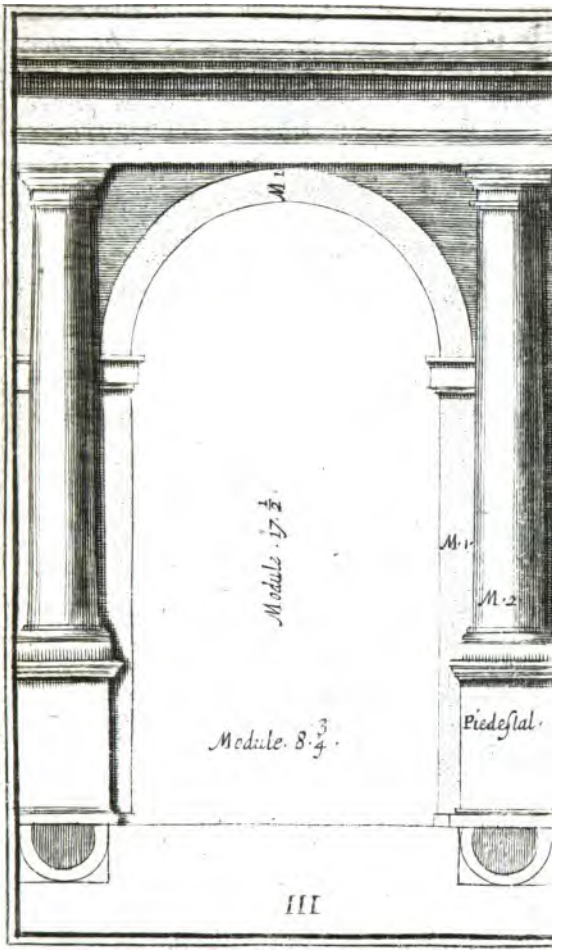

Chapit. Arch. Frise. Corniche.
Mp 4.
M 3. M 2. Mp 2.
Colomne.
M. 12.
Entre Colomne.
M 1 p.
Module. 4. 5/6.
M 1.
Base.
M 1.

I

Quant l'ordre Toscan devra estre fait sans rustique on diuisera sa hauteur entiere en 17 parties ½ chacune des quelles sera par nous appellee Module. lequel nous diuisons en 12 parties egales. & de celles ci est formé l'ordre tout entier. auec ses membres particuliers cõme l'on voit en ce dessein. qui est cotté par nombres tant rompus que entiers.

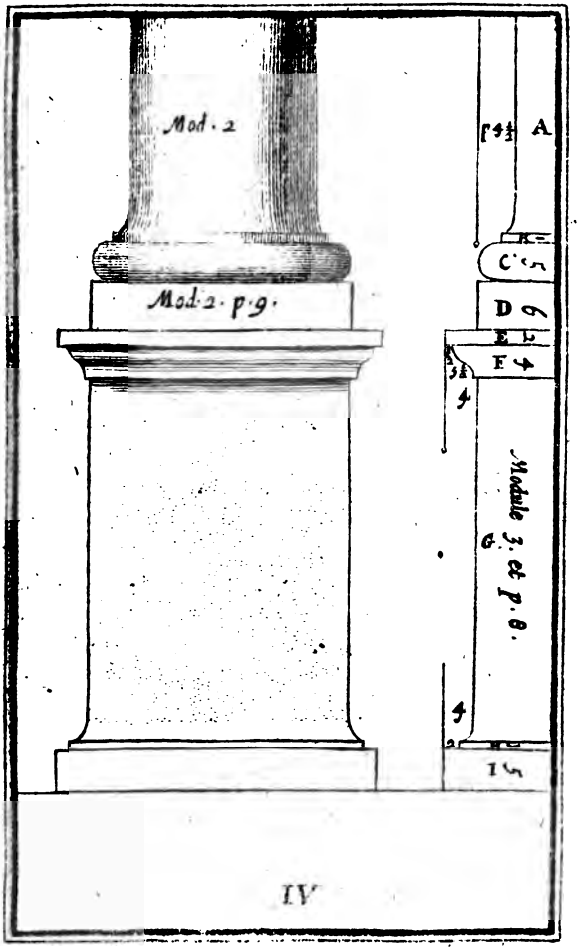

Module 13.
Module 6 1/2
II

Mais ayant a estre fait le dit Ordre auec son piedestal sa hauteur toute entiere sera diuisée en 22 parts & $\frac{1}{6}$. ce qu'on fait par ce que le piedestal requiert d'auoir en hauteur le tiers de sa colonne, auec la base & le chapiteau, laquelle estant de 14 Modules, le tiers en reuient a quatre Modules & $\frac{4}{3}$. lesquels adiouster a 17. $\frac{1}{2}$ font ensemble 22. $\frac{1}{6}$.


M.1
Module. 17 1/2.
M.1.
M.2
Module. 8 3/4.
Piedestal.

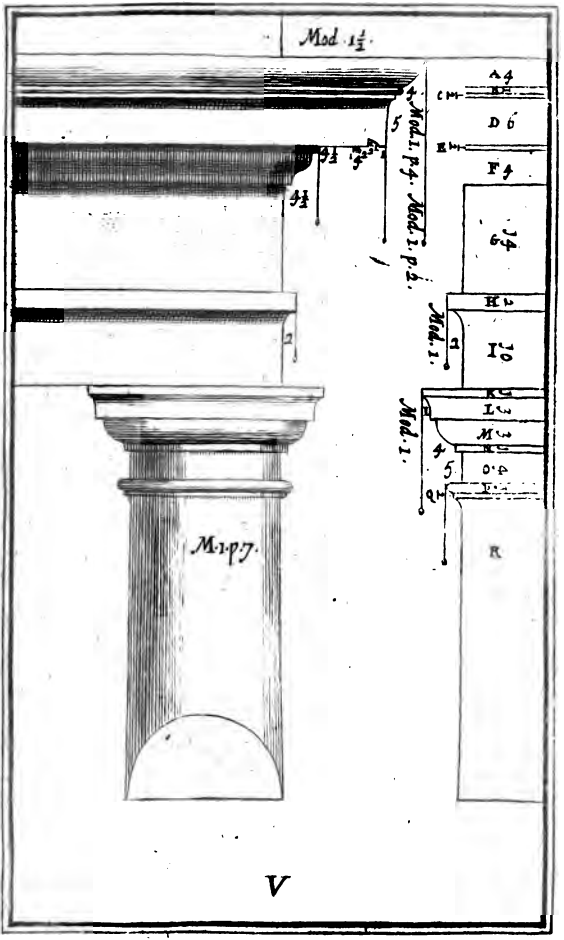
ENcor que en l'ordre Toscan il n'aduient que bien rarement qu'on y face piedestal, si estce que pour suiure l'ordre, je l'ay proposé en ce dessein, vous aduertissant en outre, qu'en tous les cinq Ordres j'ay obserué pour reigle generale, que les piedestaux auec leurs ornements doiuent estre le tiers de leur colõne auec sa base & chapiteau: tout ainsi que l'ornement d'en haut, asauoir l'Architraue frisé & corniche, en doit estre la quatriesme partie et cette Intelligence donne vn grand soulagement a ceux qui trauaillent. Par ce qu'ayant a faire l'vn ou l'autre des cinq Ordres, apres auoir determiné la hauteur qu'il doit auoir, icelle est diuisée en dix neuf parties, auec ses ornemens. Apres cela on prend de rechef la hauteur de la colonne auec sa base & chapiteau, & fait on la diuision de ses modules, selon qu'elle sera ou Dorique ou de quelque autre Ordre, & puis apres tout le reste est basti par ce module diuisé en ses parties comme l'on voit en son lieu.

A. Fust. de la Colonne. B. La ceincture plus basse de la colonne ou reigle du base. C. le Tore. D. le Plinthe. E. le Listeau, non general, & vse indifferemmẽt en tous membres semblables, soyent ils plus petits ou plus grands. F. la gueule renuersée. G. le piedestal. H. le Listeau ceincture ou reigle. I. le plinthe du piedestal.

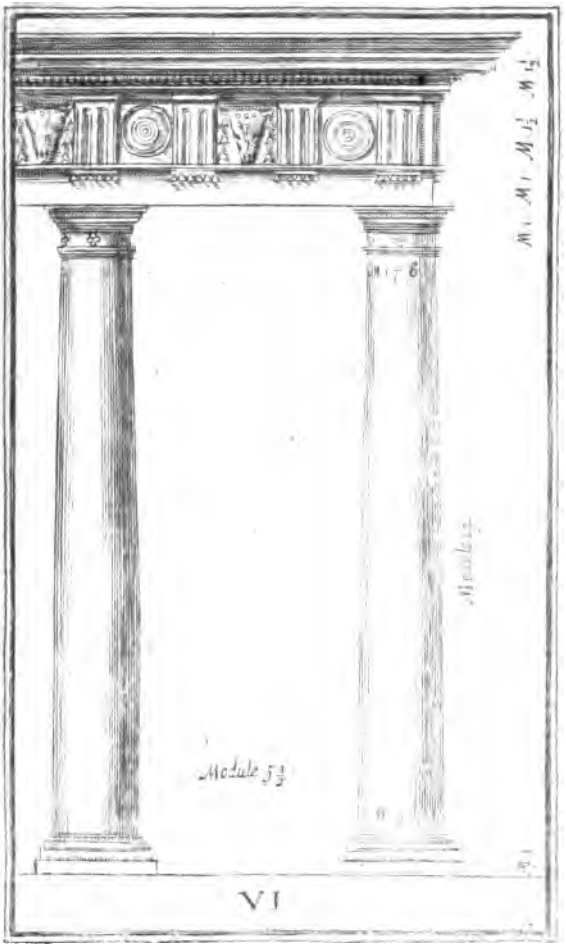

Mod. 2
Mod. 2. p. 9.
A
C
D
E
F
Module 3. et p. 8.
IV

Ayant descrit generalement ci dessus les mesures principales pour faire l'Ordre Toscan, j'ay dessiné ici & en la page precedente ces parties en gros, afin que particulieremẽt on voye la diuision de chacun des plus petits membres auec les projectures tout ensemble, & afin que la clarté du dessein auec les nombres y adjoustez soit suffisante de par soy a se faire entendre sans beaucoup de paroles, comme tout homme de consideration pourra connoistre aysement de luy mesme

A Lœuf, B Rondeau, C Listeau ou Reiglet D Couronne ou Goutierre, E Listeau F Gueule renuersée, G Frise H Liste de l'Architraue, I l'Architraue, K Listeau de l'Abaco, L Abaco, M Oeuf, N Listeau, O Frise du chapiteau, P Rondeau, Q Collier de la Colonne, R Fust de la Colonne.

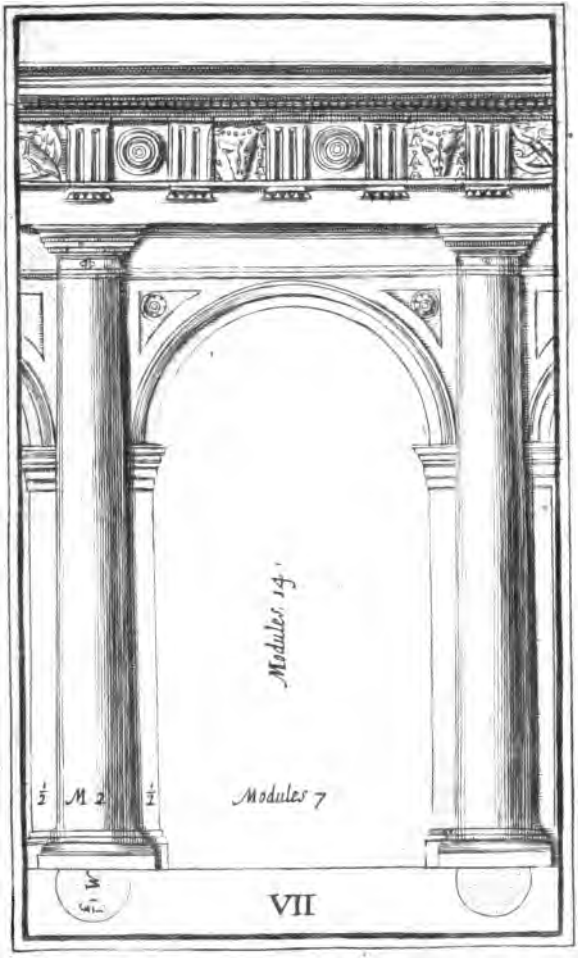

Mod. 1½.
A 4
B 1
C 1
D 6
E 1
F 4
G 6
H 2
I 10
K 2
L 3
M 3
N 2
O 4
P 1
Q 2
R
Mod. 1. p. 4.
Mod. 1. p. 5.
Mod. 1.
Mod. 1.
M. 1. p. 7.
V

LA maniere de faire la diuision de cette ordonnance Dorique sans piedestal est telle: toute sa hauteur est diuisée en 20. parties. & de l'une dicelles est fait son Module, lequel est aussi diuisé en 12. parties tout ainsi que celuy de l'ordonnance Toscane. A la Base auec l'anneau ou ceincture plus basse de la colomne sera dõné un Module. Le tronc ou fust de la colomne sans la mẽbrure plus basse sera fait de 14. modules. le chapiteau d'un module et l'ornemẽt c'est assauoir L'Architraue Frise & Corniche. sera de 4. modules. qui est le quart de la Colõne auec sa base & chapiteau. cõme desia nous auons dit ci dessus que l'Architraue doit estre d'un la frise d'1 ½ & la corniche d'1 ½ qui adiousteez ensemble sont 4. & sommez auec les autres reuienent a 20.

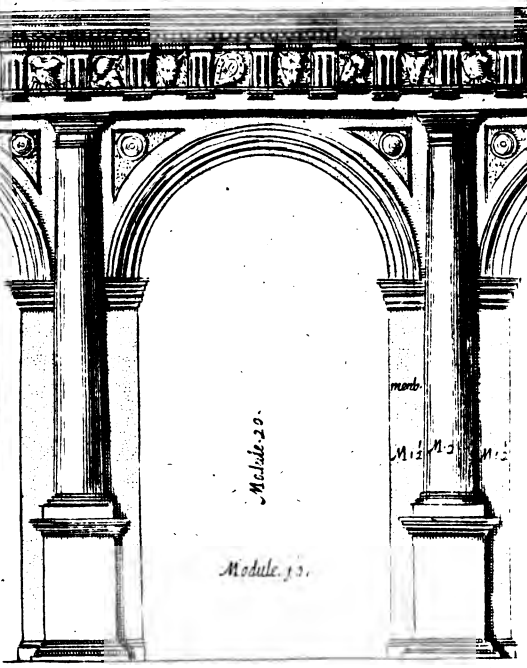


VI

QVand on voudra faire l'ornement des Galeries ou portiques selon l'ordonnance Dorique, il faudra (comme il a esté dit) diuiser la hauteur en 20. parties, & en former le Module puis apres on partagera en telle maniere les largeurs, que de l'un jusques a l'autre Pilastre, il y aye 7. modules, & que les pilastres en ayent trois. Car ainsi les largeurs et hauteurs se trouueront diuisées en leur proportion et la lumiere des vuides aura la hauteur double de sa largeur, & viendra juste la distribution des Metopes & Trigliphes, comme l'on voit. Reste seulement a considerer, que la colonne doit sortir hors du Pilastre le tiers d'un module plus que sa moitié. Ce qui se fait afin que les Projectures Saillies ou larmieres des Impostes ne passent la moitié des colonnes. Et sera ceci une reigle vniuerselle en tous cas semblables de touts les ordres.

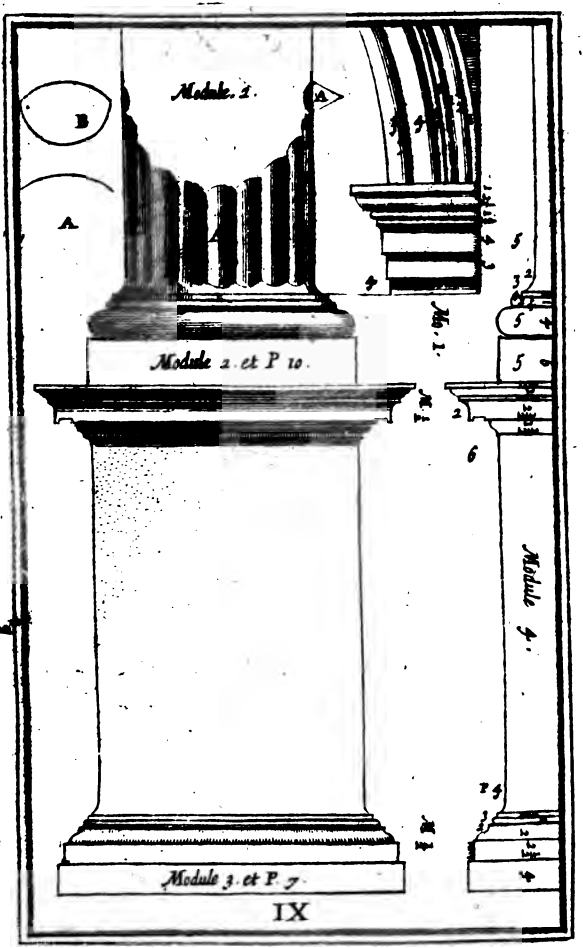

Modules 14.
½ M 2 ½
Modules 7
VII

AYant a faire des Galeries ou portiques selon l'ordonnance Dorique auec leurs Piedestaulx, la Colomne deura estre diuisée en 25. parties & ⅓. & de l'une d'icelles sera fait le module. la largeur de l'un jusques a l'autre Pilastre sera terminée par 10. Modules, & la largeur des pilastres par 5. Car c'est ainsi que viendront a estre justes les distributions des Metopes & Trigliphes, & le vuide des arcs proportionne requerant que la hauteur soit double de la largeur, laquelle est de 20. modules, comme l'on peut voir.



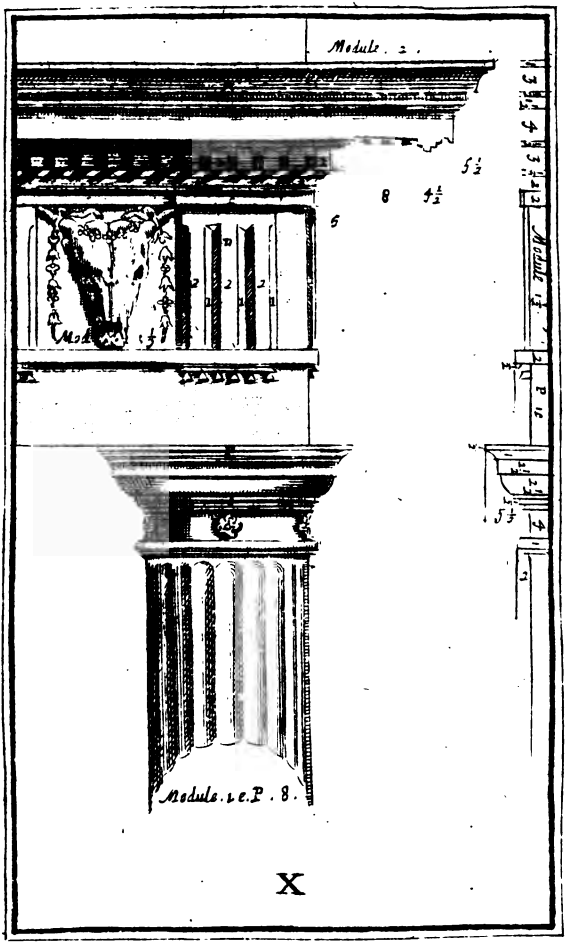
VIII

LE Piedestal de l'ordonnance Dorique doit auoir 5. modules $\frac{1}{2}$ en hauteur, l'Imposte de l'Arc ci dessus designé 1. module, & ses membres particuliers seront repartis selon que portent les nombres ici notez. A. les canaux ou rayons de la Colomne. B. la ceinture plus basse de la Colonne laquelle doit estre entendue ainsi par tout les Ordonnances. C. Rondeau, ou baguette.



IX

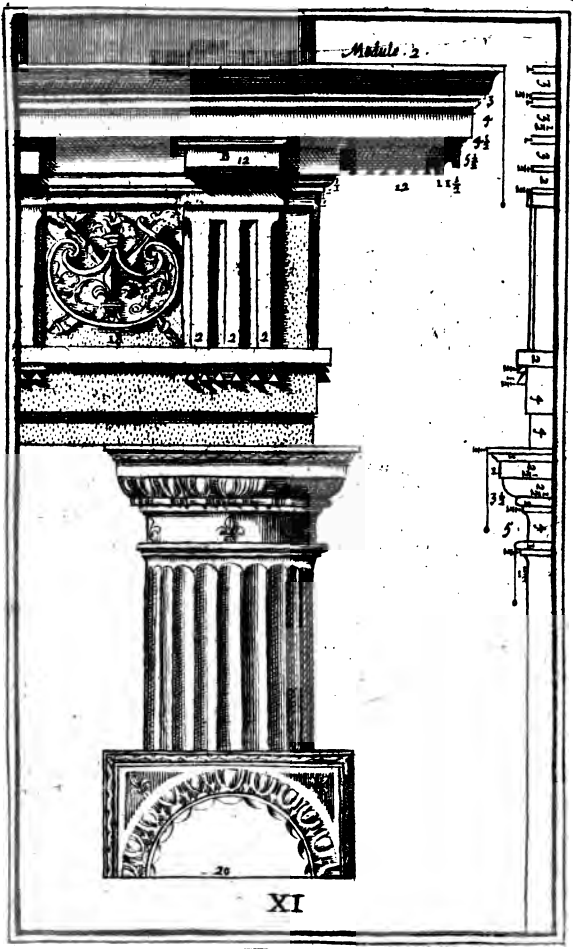
CEtte partie d'ordonnance Dorique est tirée du Theatre de Marcellus a Rome, comme je l'ay allegué dans l'Auant-propos par maniere d'exemple. & estant dessinée elle retient cette mesme proportion. A. Tuyau, B. Dentelets, C. Chapiteau du Triglife, D. Triglife, auquel les parties creusées interieurement sont appellées canaux ou rayons. & l'espace quarré de la frise qui reste entre l'un & l'autre trigliphe est appellé Metope. E. Goutes ou elochettes. F. Cymaise G. Annelets, ceintures ou listeaux.



X

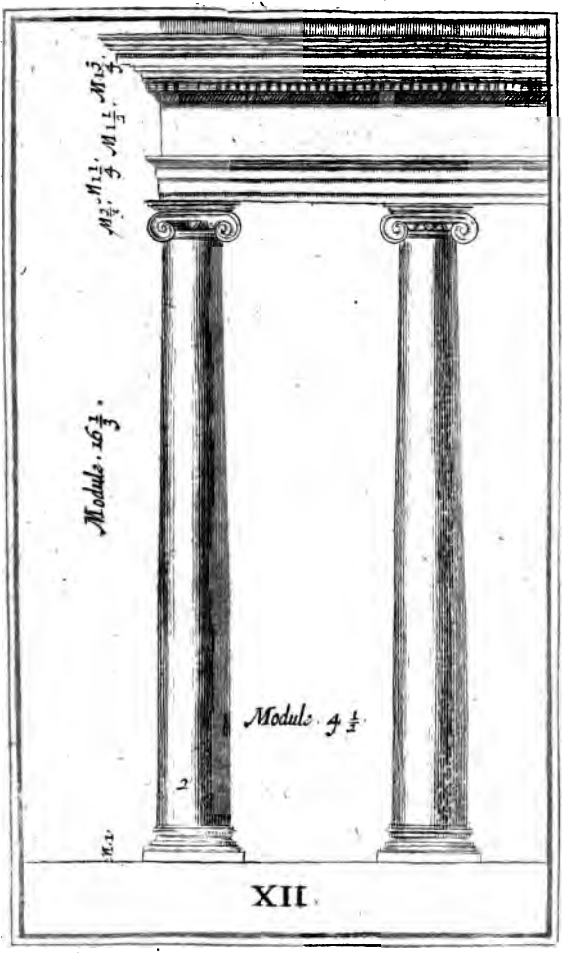
CEtte autre piece d'Ordonnance Dorique est tirée de diuerses reliques d'entre les Antiquités Romaines il s'en fait vn tel composé qu'en le mettant en œuure ie l'ay trouué venir bien a propos.

A. Gueule droite B. Modillon ou Modelle, nom du quel ils sont appelles tous encor qu'ils ayent la forme differente moyennant qu'ils facent l'office de soustenir la corniche qu'ils ont chargée C Pesons. ou piroüet.

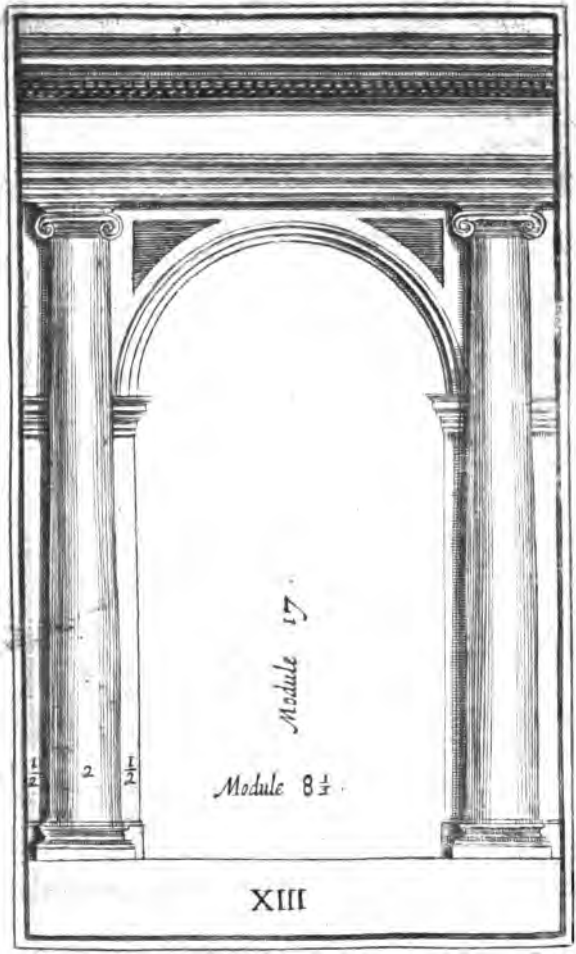


XI

Voulant faire l'ordonnance ionique sans Piedestal, toute la hauteur doit estre diuisée en 22 ½. parties, & de l'une d'icelles est fait le module, lequel est reparti en 18. Ce qui se fait d'autant que celte ordonnance, pour estre plus gentile que la Toscane & Dorique, a aussi les diuisions plus menues : Sa colomne doit auoir 18. modules, y compriſe la baſe & le chapiteau, l'Architraue 1. module & ¼. la Friſe 1 ½. module, la Corniche 1 ¾. module Nombres qui adjouſtez enſemble rendent l'Architraue, Friſe & Corniche de 4 ½. modules, qui eſt le quart de la hauteur de la Colomne.

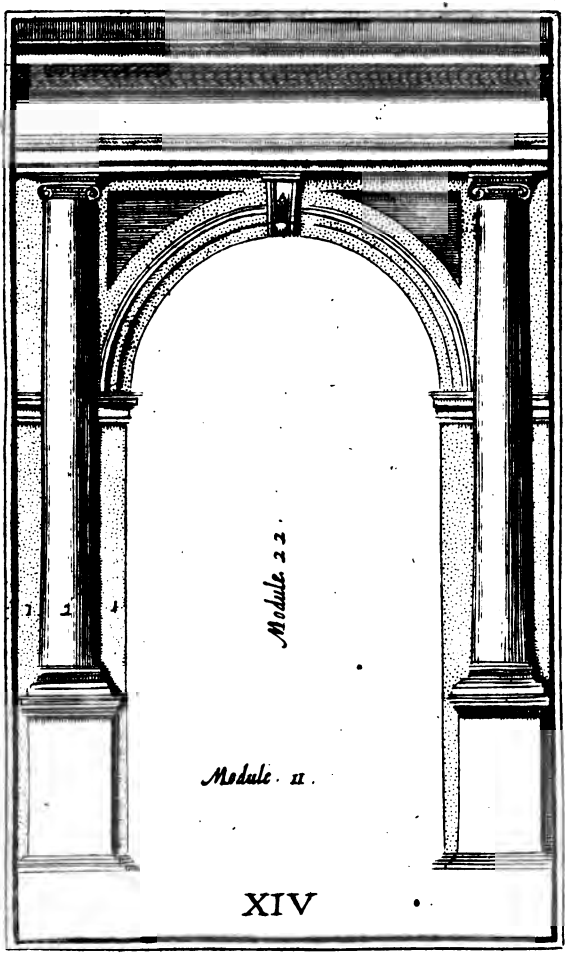

Module. 16 1/3.
Module. 4 1/2.
2
XII.

QVand on voudra faire des Galeries ou Portiques selon l'ordonnance Ionique les Pilastres se feront de la grosseur de 3. modules, & la largeur du vuide sera de 8. modules, ½ la hauteur de 17 qui sera le double de la largeur. Regle qu'on doit observer fermement en tous arcs de semblables ornemens, toutes & quantes fois que la grande necessité ne nous contraint de faire autrement.



XIII

MAis quand on voudra faire des Galeries ou Loges selon l'ordonnance Ionique auec les Piedestaux, toute la hauteur est diuisée en 28. ½. parties, le Piedestal auec son ornement ayant 6. modules, qui est le tiers de la Colonne auec sa base & chapiteau. cõme nous auons dit que cela s'obserue en toutes les ordonnances. La largeur du vuide sera de 11. modules, la hauteur de 22. La largeur des Pilastres de 4. modules, comme lón voit noté par nombres en ce dessein.



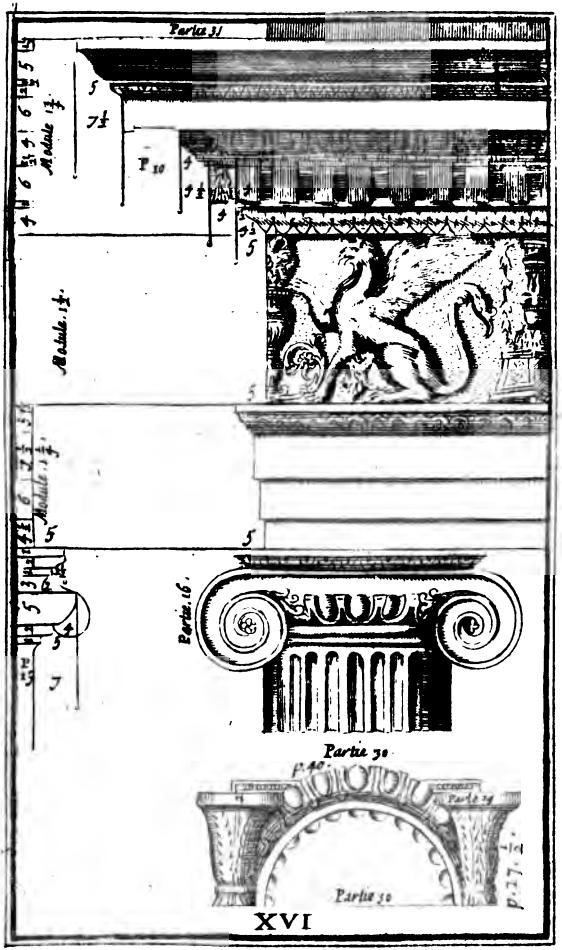
XIV

LA Corniche de l'impoſte d'enhaut eſt d'vn module, ſa projecture d'vn tiers, les membres particuliers on les peut apprendre par les nombres, comme auſſi ceux du Pied deſtal & de la baſe.

A. Scotie ou creux d'enhaut. B. Aſtragales ou rondeaux. C. Scotie, ou creux d'embas.

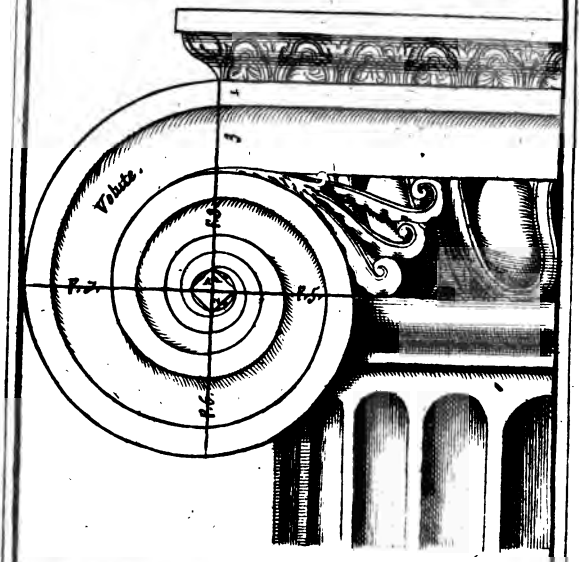
A
B
C
Module. 1.
M. 1.
P. 10.
Cette hauteur de 5. Modules doit être
P. 8.
M. 1.
XV

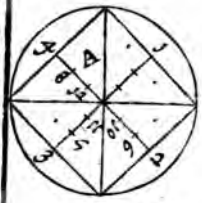
LA maniere de faire le Chapiteau Ionique, combien qu'elle ſoit pourtraitte en ceſte figure auec le plan & profil, ſi eſtce que pour plus claire intelligence il faut tirer deux lignes perpendiculaires, ſeparées deux modules l'une de l'autre, qui paſſent par le centre des yeux de la volute, & ſont appellées Cathetes. toute la volute doit auoir la hauteur de 16. parties de module, dont les huit demeurent au deſſus de l'œil (lequel tient deux parts) & les autres ſix reſtent au deſſous. La maniere de faire ces volutes eſt pourtraitte au feuillet ſuiuant, ou auſſi ſera expoſée brieuement (tant que l'eſpace le permettra) la maniere ſelon laquelle on procede.


Partie 31
Module 12.
Module 12.
Module 12.
Partie 16.
Partie 30
P. 46.
Partie 24
Partie 30
P. 27.

XVI

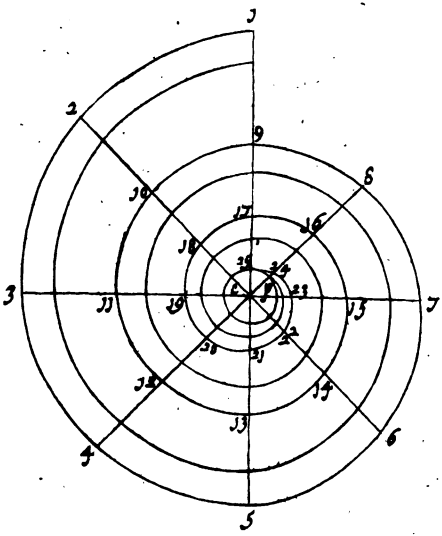
Ayant tiré la cathete O.M. de cette premiere volute & vne auec. ligne a leſquerre par le centre de l'œil, l'œil ſera diuiſé ainſy qu'on le voit en la figure A. puis on commance au premier point ſigné I. auec le compas pour faire le premier quart de cercle & du point 2. on tire le ſecond quart de cercle & pourſuiuant ainſy les trois tours viennent a eſtre acomplys. or pour faire la largeur de la bande R. il ne faut apres luy auoir donné au commencement la largeur du filet que tourner ſur les meſmes centres dits cy deſſus & par ce moien la bande ce reſtreſſira a meſure quelle aprochera du centre.





XVII

QVe si vous voulés faire la volute par laquel. maniere cy dessoubz repritée soit tirée la ligne 1.5. apellée cathete laquelle aura la hauteur de 16. partz dun module dont les 9. demeureront au dessus du centre E. et les 7. au dessoubz puis sur ce centre soit faict une diuision en 8. parties esgales ainsy quelle vous est repritée par les 8. lignes qui partent du cẽtre puis soit faict apart le triangle B.C.D. de sorte que la ligne B.C. ayt 9. partz dun module et la ligne C.D. 7. et soit larc G.C. diuisé en 7. parties esgales dõt lune demeure pour le demy diametre de lœil, et le reste G.N. soit diuisé en 24. parties esgales continuées jusques en la ligne B.C. qui les rendra jneſgales, soit pris en apres la grandeur B.C. et raportée de E.A 1. et marquée puis de C.A. 2. et raporter de E. a 2 et marqué puis C. 3. raportés a E. 3. et continuer ainsy sur les 8. diuisions jusques a ce quon y ayt marqué tous les 25. pointz de B.C.I. mais pour mener maintenant vne circõference qui du point marqué 1. aille au point marqué 2. soit ouuert le cõpas de 1. a E. et estant poze ferme sur 1 soit meu le compas pour descrire la portion de circonference cy ponctuée puis soit la mesme ouuerture du compas raportée sur 2. jusques a ce questant meu il face vne section en p. qui sera le centre pour descrire la circonference de 1 a 2. et soit ainsy continué pour faire 2. a 3. et pour les autes parties de circonference.


1
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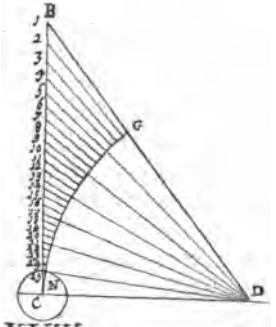
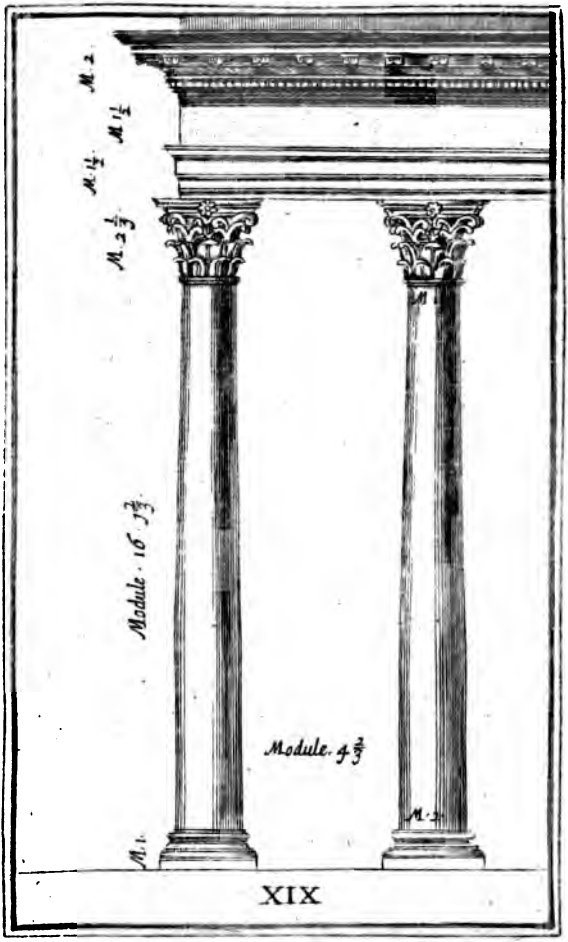

B
G
N
C
D

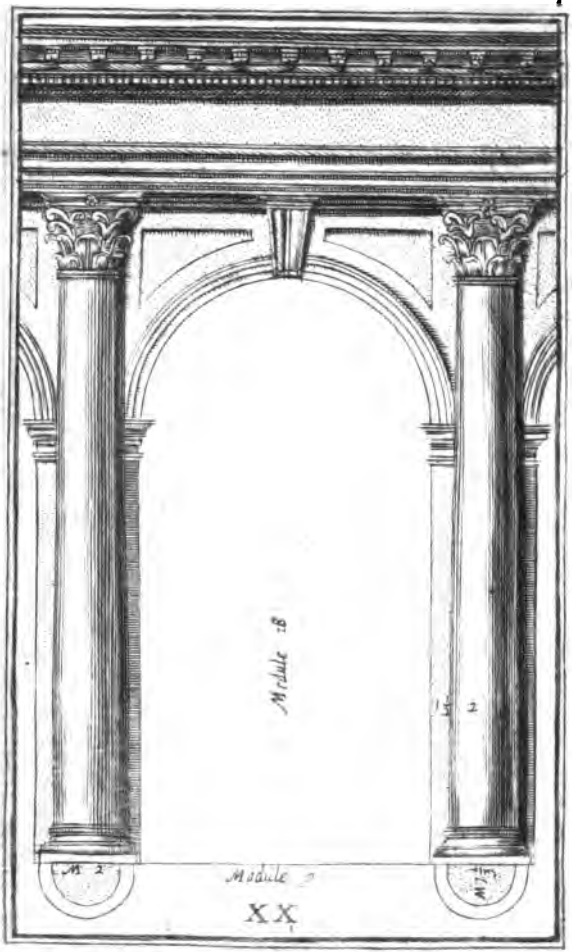
XVIII

POur faire cette ordonnance Corinthienne sans Piedestal, toute la hauteur est diuisée en 25. parties, & de l'une d'icelles est fait le module, lequel est diuisé en 18. parties, tout ainsi que celuy de la Ionique les autres diuisions principales se voyent en la figure & la largeur de l'une a l'autre Colonne doit estre de 4. modules & ½ tant afin que les Architraues d'enhaut ne soyent surchargées, que pour accommoder les modelles d'enhant en la Corniche de telle sorte qu'ils respondent droitement au milieu des Colomnes en leur compartiment egal.

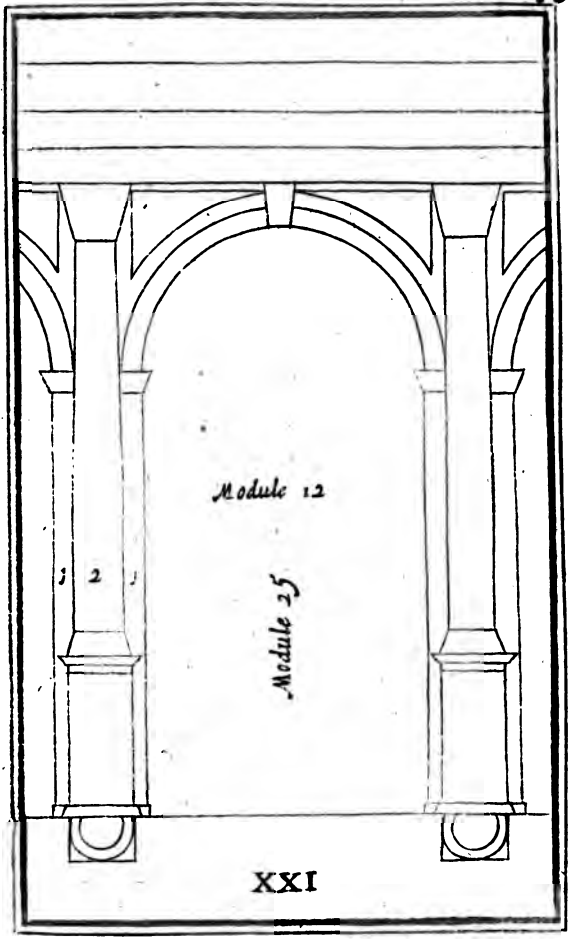


XIX

VOulant faire des Arcs de Galeries selon cette ordonnance Corinthienne on y doit proceder comme il est ici noté par nombres tellement que les vuides tiennent 9. modules de largeur & 18 de hauteur, les Pilastres seront de 3 modules.


Module :8
Module
XX

Mais ayant a faire des Portiques ou Galeries auec piedestaulx, toute la hauteur sera partie en 32. de l'une desquelles on fera le module, 12 dicelles sera la largeur du vuide. & 25 sa hauteur: Et quoy. que cela passe les deux carrez, si est il conuenable en cette ordonnance. a cause de la gentillesse. Les Pilastres se feront de quatre modules. comme il est noté.



XXI

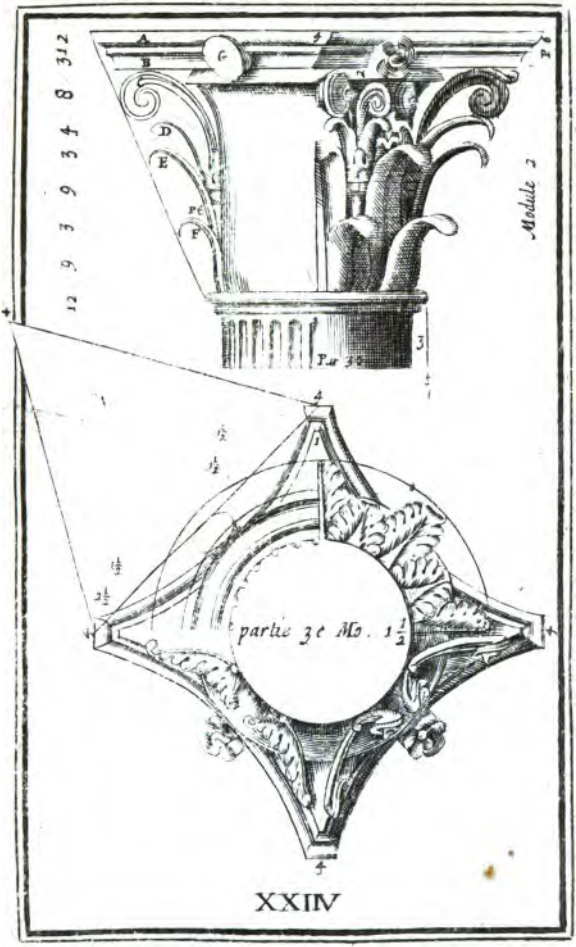
SI le Piedestal de cette ordonnance Corinthienne est le tiers de la colonne. il tiendra six modules et ⅔. mais on le pourra bien faire de sept modules pour plus grande solidité, fort conforme et conuenable a cette ordonnance: et aussi. afin que le Piedestal. sans la cimaise et la base. reuienne a deux quarrez. comme l'on pourra voir par les nombres. le reste, c'est a sauoir la cimace et le basement. puis qu'ils sont notez par le menu. comme aussi l'impost de l'arc. il n'est pas besoin d'en escrire d'auantage.

A. le tore ou baston d'enhaut, B. le tore ou baston d'enbas.

Module 1
B
Partie 14
Module 5 parties 12
Partie 12.

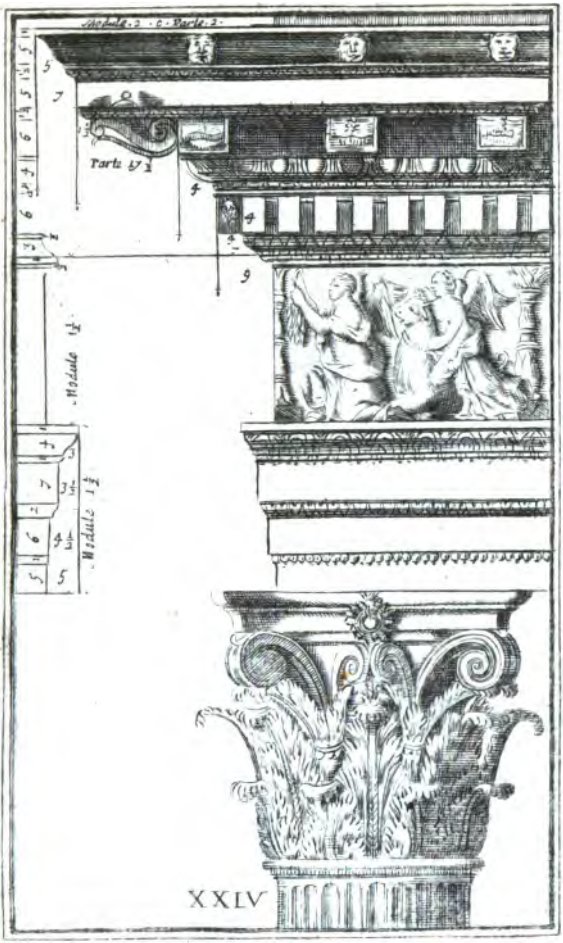
AV plan & profil de ce chapiteau Corinthien on pourra connoistre toutes ses mesures. Par le plan on mesure les largeurs, y faisant vn quarré duquel la ligne Diagonale sera de 4 Modul. & de l'vn de costez de ce quarré on fera vn triangle, selon qu'on voit en la figure, & la pointe du compas on la ferme au coin signé +, & lon tire le creux de l'Abaque. Au profil est prise la hauteur de ses feuilles, tigettes & Abaque. et l'estendue des feuilles et tiges est prinse par la ligne qui naist de la pointe de l'Abaque jusques au rond de la colomne comme l'on voit par le dessein du profil. le reste auec peu de consideration qu'on y apporte, sera aisement entendu.

A. & B. sont appellez ensemble l'Abaque du chapiteau. mais pour plus claire intelligence A. est pris pour la cimaise de l'Abaque, C. Tiget. D. feuilles moindres, E. Feuilles du millieu. F. feuilles de dessous, G Fleur.

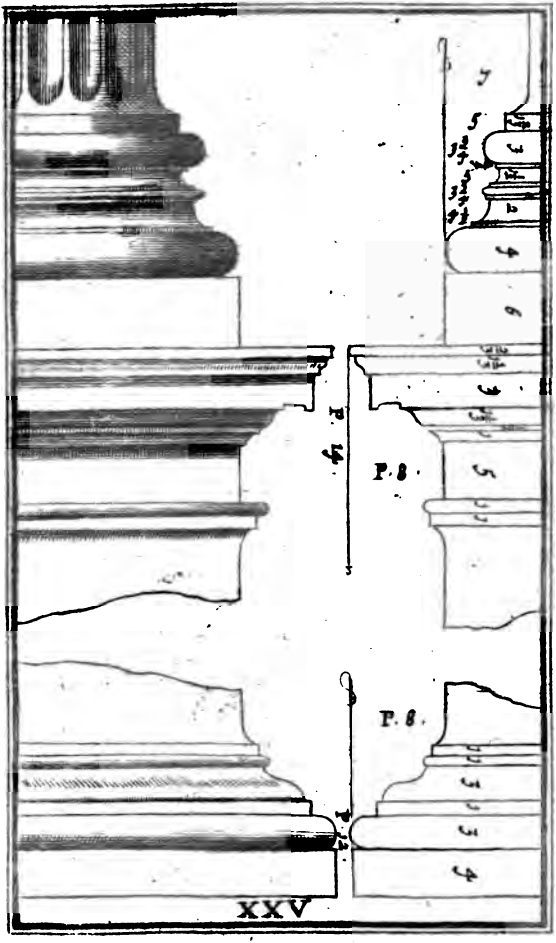


XXIIII

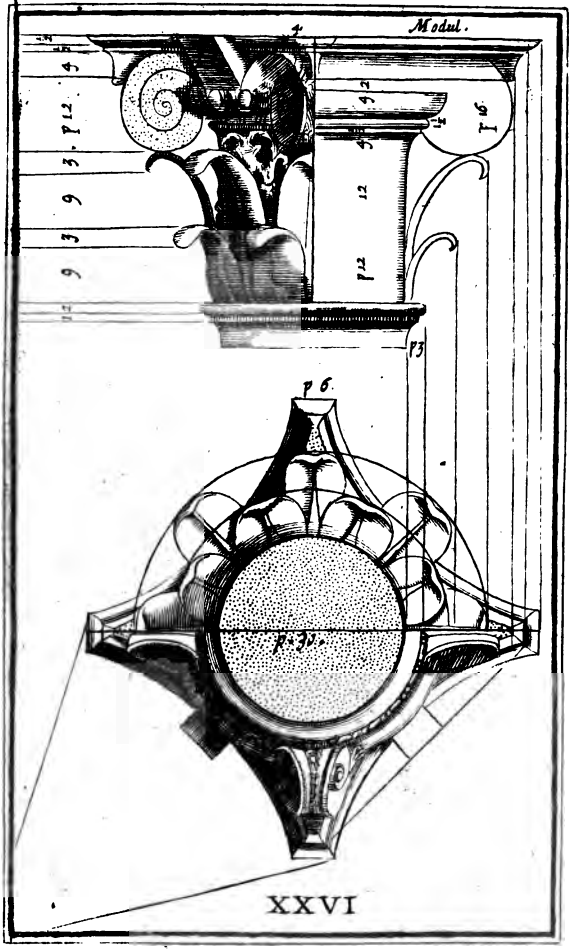
CEtte Corniche Corinthienne eſt tirée de diuers endroits a Rome & principalement du Pantheon ou temple de la Rotonde & de trois colonnes qui ſont au marché Romain, et confronter les membres principaux d'icelle. jay mis en auant ſa reigle (ne m'eſloignant rien qui ſoit des anciennes) & icelle reduite a telle proportion, qu'il en vient vn Modiglion au millieu de la colonne & que ſes œufs dentatdes, arcquets & ſpondiles reſpondent droitement l'un a l'autre, auec vn ordre ſi ſoigneux que l'on pourra voir pour donner a connoiſtre ſes meſures ſuppleront les nombres faicts par modules & parts de modules, le module party en 18. comme dit a eſté.


Module 15
Module 15
XXIV

CE Piedestal composé garde les proportions du Corinthien, & n'y a autre difference de membres qu'en la cimaise & la base, comme l'on peut connoitre. et d'autant que l'ornement composé a mesme proportions auec le Corinthien, j'ay jugé n'estre necessaire d'en faire des colonnes & arcs a part, me raportant aux Corinthiens. Seulement ay je montré la diuersite de la base & du chapiteau, & autres leurs ornements. comme l'on pourra voir en ses lieux.

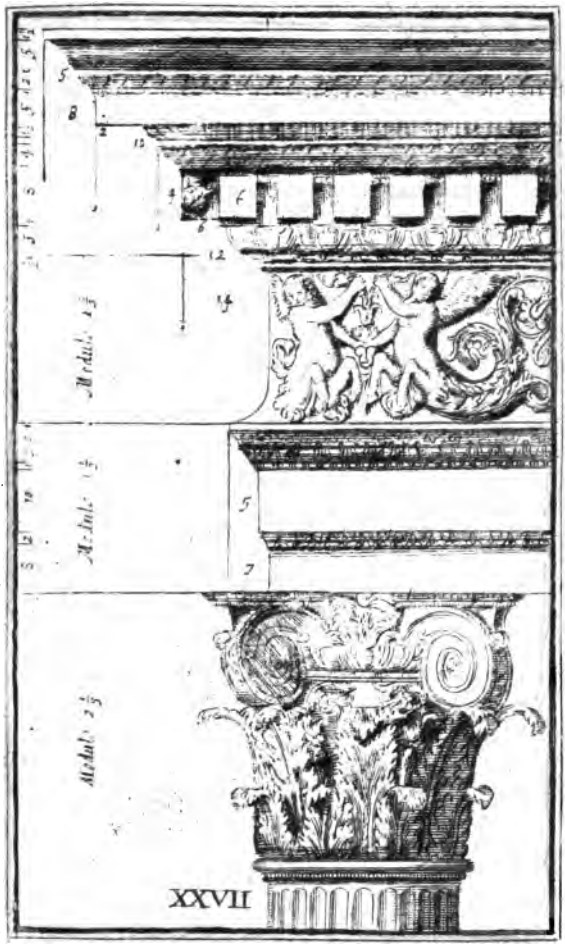

P. 8.
P. 8.
P. 8.
XXV

CE plan & profil du Chapiteau cõposé a mesme procedure auec celuy de l'ordonnance Corinthienne descrite ci dessus, il n'y a que ceci de difference: c'est assauoir, la ou le Corinthien a ses tigettes, ce Composé a les volutes faites de mesme maniere auec celles de la Ionique les anciens Romains empruntans vne partie de la Ionique, et vne autre du Corinthien, ont fait vn tel Composé, pour vnir ensemble en vne seule partie tout ce qu'ils pouvoyent recouurer de beauté...



XXVI

CEtte pièce d'ordonnance Composée, asçauoir le Chapiteau Architraue Frise et Cornice est aussi tirée de diuers lieux d'entre les Antiquités de Rome et reduite a proportion comme il a esté dit de la Corinthienne. Et qu'estant noté par nombres se montre assez de soy mesme.


Module 1 3/5
Module 1 1/2
Module 2 3/5
XXVII

Entre les Antiquitez de Rome se trouuent presque infinies varietez de chapiteaux qui n'ont point de propres noms mais peuuet estre cõpris tous ensemble sous ce nom general de Composé et suiuẽt aussi les mesures principales des autres cõposez deriuez de l'Ionique et Corinthie tant seulemẽt il est bien vray qu'en aucuns d'iceux lõ verra des animaux en lieu de tigettes en autres des cornes d'abõdance, et en autres plusieurs autre choses selõ qu'il est venu apropos aux Maistres en leurs desseins, ce qui se peut juger par le proset jà dessiné, lequel ayant quatre Aigles en lieu de tigettes et en lieu de frutage quatre faces de Iupiter auec la foudre au dessous dõne a connoistre aisemẽt qu'il est de quelque exemple cõsacré a icelluy Iupiter tout ainsi l'on peut dire de cet autre qui a quatre Grifons en lieu de tigettes et quatre Aigles au millieu d'iceux auec vn chien en leurs ongles qu'il ait esté appropriè a quelque autre de leurs Idoles sa proportiõ hors mis les animaux est semblable aux Corinthiens.

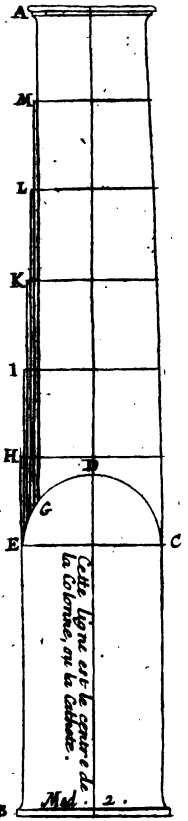
Cette base, Vitruue lib 3. chap 3. la nõme Atticurga, cõme estant trouue originairemẽt et mise en œuure par les Antiens De nostre tẽps on vse de la mettre en œuure indifferemẽt sous le Corinthien, Cõposé Ionique et Dorique toutesfois elle a plus d'alliance auec le Cõposé qu'auec nul autre, et est aussi tolerable en l'Ionique qu'ad on ne se sert de la propre base d'icelle. mais sous les autres ordonances je la jugerois du tout impertinente et en produirois beaucoup de raisõs mais je ne veux m'amuser a parler auec vne si grande licẽce de chose passee. il suffira en mesme ordre que ci dessus d'en monstrer la partitiõ qui n'aist du module lequel est diuisé en 18. parties tout ainsi que ceux del'Ionique et Corinthien

[illegible]

Module 2 Parte 14

XXVIII

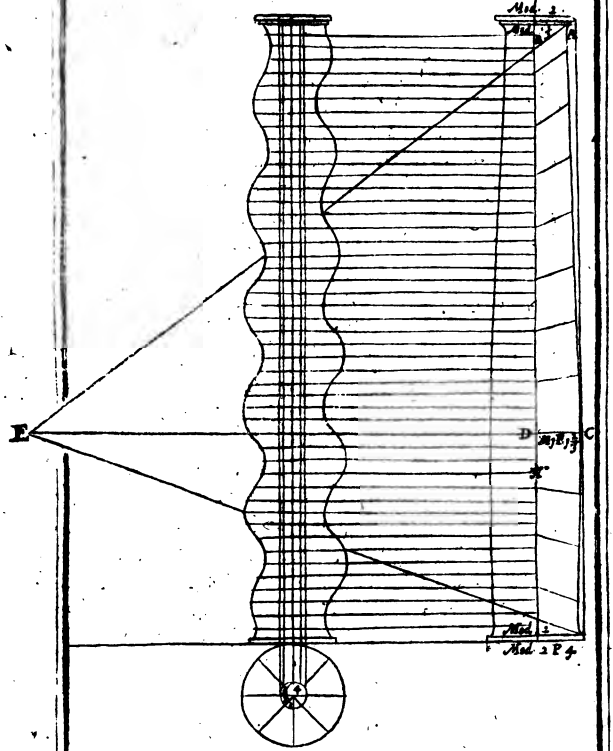
LA diminution des Colomnes se fait en plusieurs façons dont voicy les deux meilleures. pour la premiere soit la colomne A.B. a diuiser du tiers en hault soit au point E. fait le demy cercle E D.C. et du point de la diminution A soit abessée perpendiculairement la ligne A.G. soit puis apres l'arc G.E. diuisé en tant de parties esgales qu'on voudra comme en six et soit E.A. diuisé aussy en autant dalles parties esgales et ou les lignes perpendiculaires et orisontales se couperont soit menée la ligne L.H. I.K.L.M.A. qui donnera la diminution comme on la desire et de cette facon on s'en peut seruir au Toscan et au dorique.


A
M
L
K
I
H
D
G
E
C
B
Cette ligne est le centre de la Colonne, ou la cathete.
Mod. 2.

XXIX

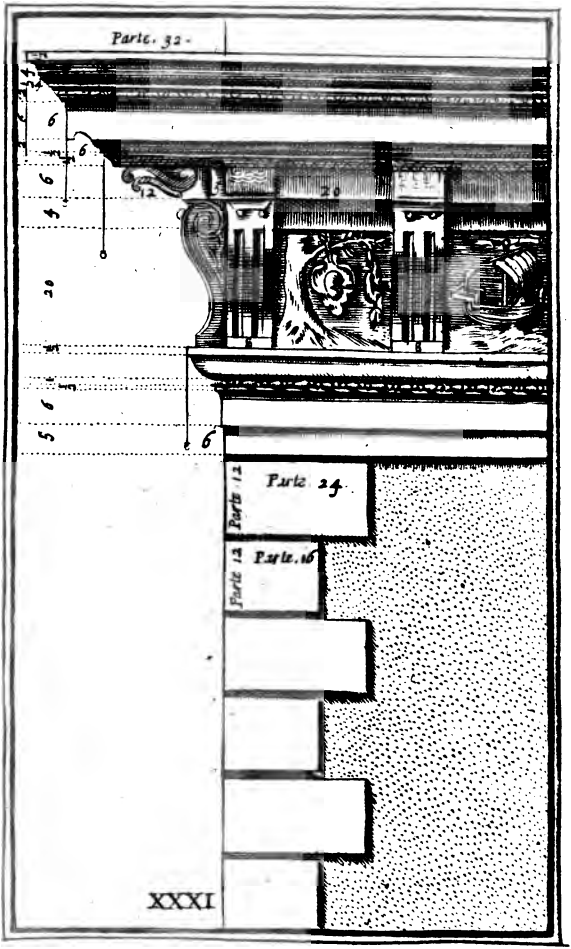
Quand a cette autre façon je l'ay trouuée de moy mesme et quoy quelle soit moins conçüe elle est neantmoins facile a conceuoir par les lignes, je diray donc quayant resolu les mesures de la colone on doit tirer vne ligne a lonplus en amõçant par C. et continuant par D. puis reportant la mesure C. D. au point A jusques a ce quelle coupe la perpendiculaire au point B. et que AB soit continuée jusques en E. de la on pourra tirer tant de lignes qu'on voudra qui partiront de la perpendiculaire & iront a la circonference de la colonne sur lesquelles a pliquant la mesure C.D. on trouuera tant en haut quen bas l'enfleure de la colonne et cette maniere peut estre apliquée a l'Ionique Corinthien et Composé.

Que sy lon veut faire ces colonnes torses a la maniere de celles qui sont a S. Pierre de Rome on en doit faire le plan comme il est icy marqué, et ce petit cercle du milieu montre de combien elle est torse lequel estant diuizé en 8. parties et ayant tiré ces 4. lignes parallelles a la perpendicalaire on diuisera toute la colonne en 48. parties et lon fermera cette ligne spirale par le meilieu qui sert de centre a la colõne sur laquelle on reportera la grosseur de la colonne droitte ligne par ligne comme il paroist icy, mais il faut prendre garde que les 4. nombres 1. 2. 3. 4. marques sur le plan sont pour seruir seulement jusques a la premiere demy montée par ce que le commancement part du centre et quil faut de la en haut suiure par le tour du petit cercle jusques a la derniere demy montée den haut ou il se fault encore seruir des quatre premiers poinlz denbas.



XXX

CEtte Corniche que j'ay mis en œuure par pluſieurs fois pour l'accompliſſement des frontiſpices, et trouué eſtre fort aggreable, quoy qu'elle ſoit de mon inuention, ne m'a pourtant ſemblée impertinente pour la ſatiſfaction de ceux qui ſen voudront ſeruir, d'eſtre miſe ici ſur la fin de ce petit œuure. A proportion auec le frontiſpice eſt telle, que tout la hauteur eſtant diuiſée en onze parties, il en demeure vne pour la Corniche et dix pour le frontiſpice. le reſte ſe voit aſſez clairement.


Parte. 32.
Parte 12
Parte 24.
Parte 12
Parte. 16
XXXI

PLan et ornement du dessous de la corniche de lór-
dre Dorique comme elle est sy deuant a la figure jo

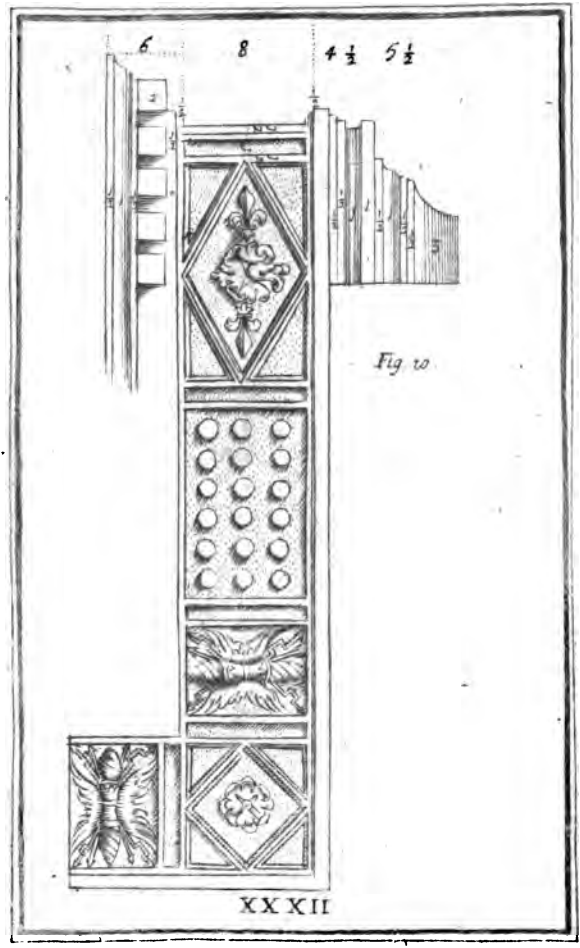
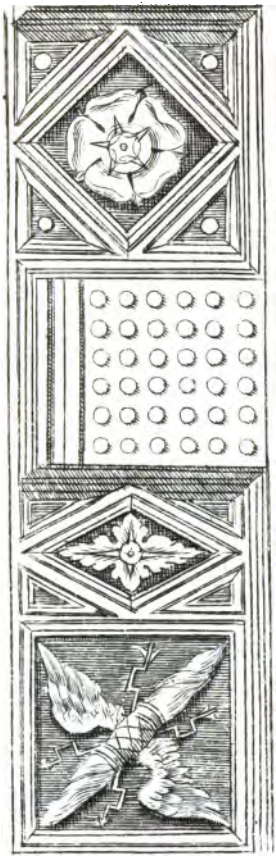


Fig 10

XXXII

Plan et ornement du dessous de la corniche de lordre Dorique comme elle est cy deuant a la figure yy.



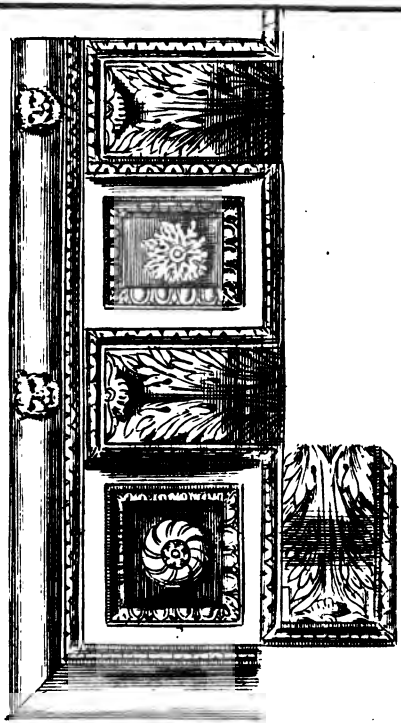
XXXIII

A.. C Orniche de l'jmpoſte Corinthien.

B.. C Orniche de l'jmpoſte Ionique

A
5
4
3
P 6
B
7 ½
6
P 6

Plan et ornement du dessous de la Cor= niche de l'ordre Corinthienne.

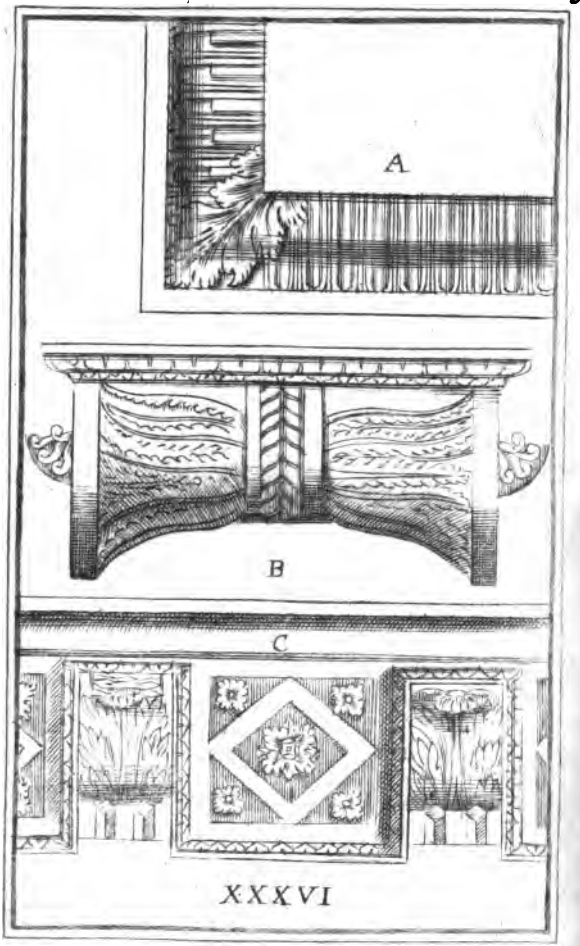


XXXV

A.. Plan et ornement du dessous de la Corniche de l'ordre Composé.

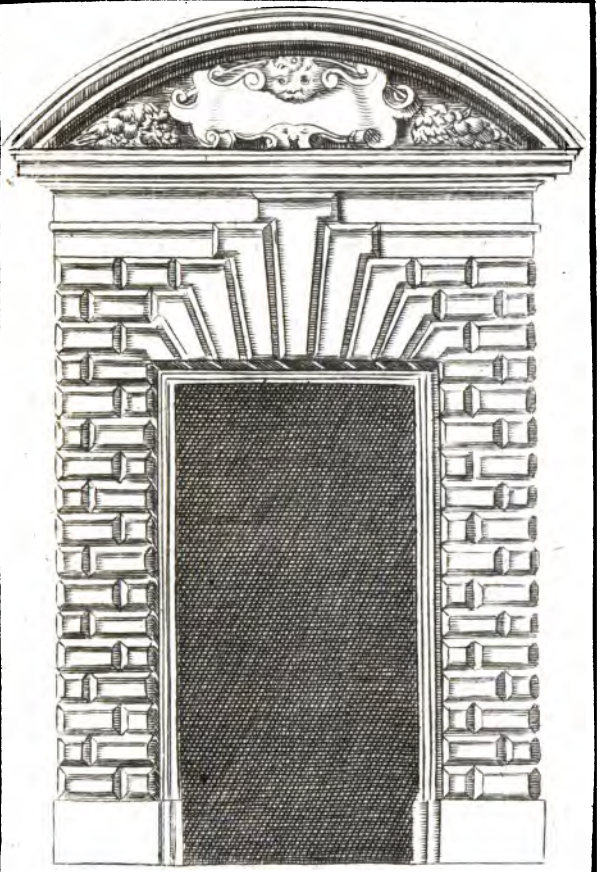
B.. Profil du Chapiteau de l'ordre Ionique

C... Plan et ornement du dessous de la Corniche de la figure 31.me

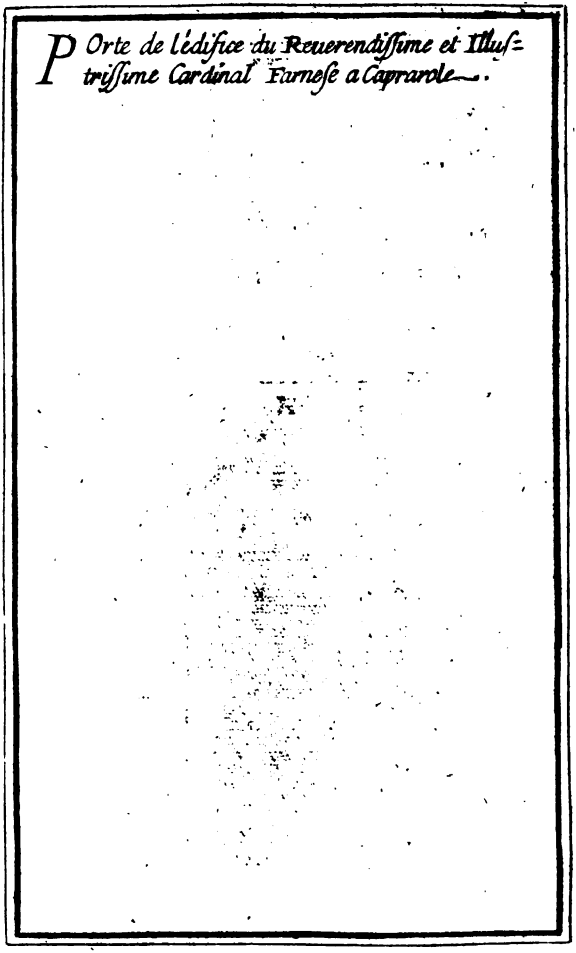


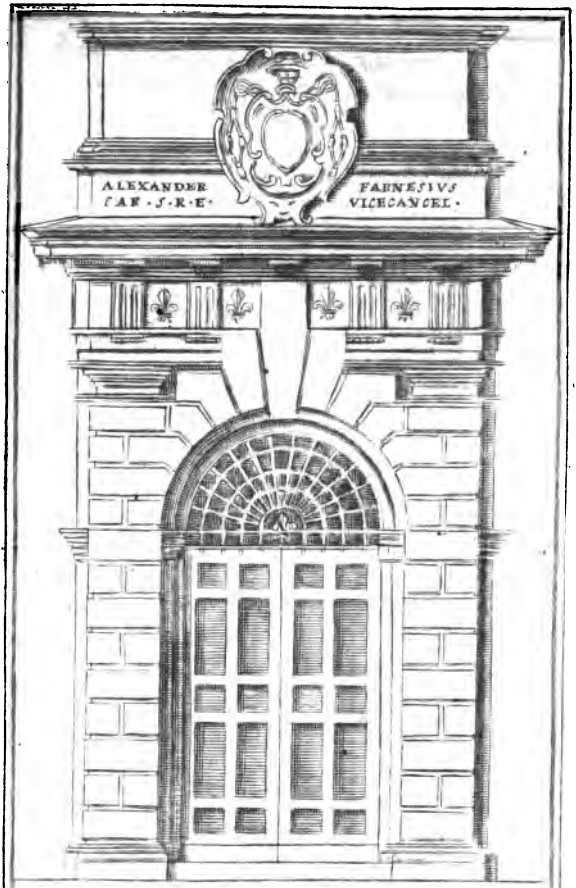
XXXVI

CEtte Porte eſt d'œuure Ruſtique et ſont ſi bien compoſées enſemble les pierres d'icelle, qu'encor qu'il n'y eut ni chaux ni autre mixtion, ſi ſont elles ſuffiſantes de regir tout Edifice quelque grand qu'il ſoit.


1 2 3 4 5 6
XXXVII
12 P.

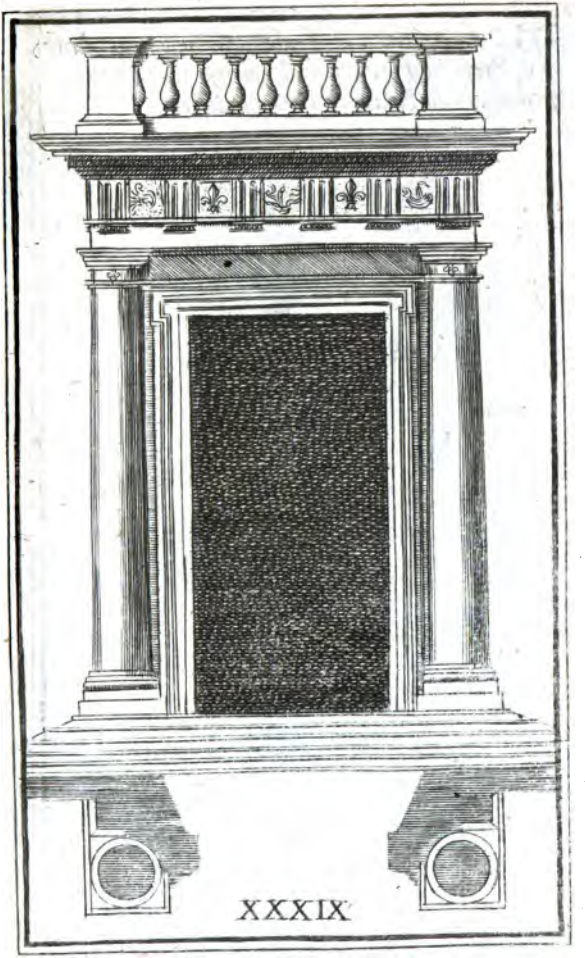
Porte de l'édifice du Reuerendiſſime et Illuſtriſſime Cardinal Farneſe a Caprarole.





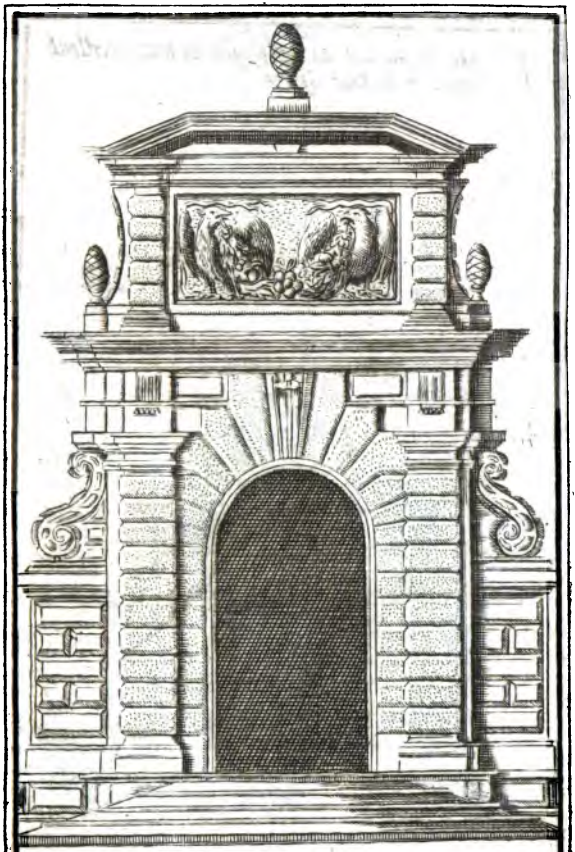
XXXVIII

Porte dessinee au seruice de l'Illustrissime & Reuerendissime Cardinal Farneſe pour l'entrée principale du Palais de la Chancelerie.



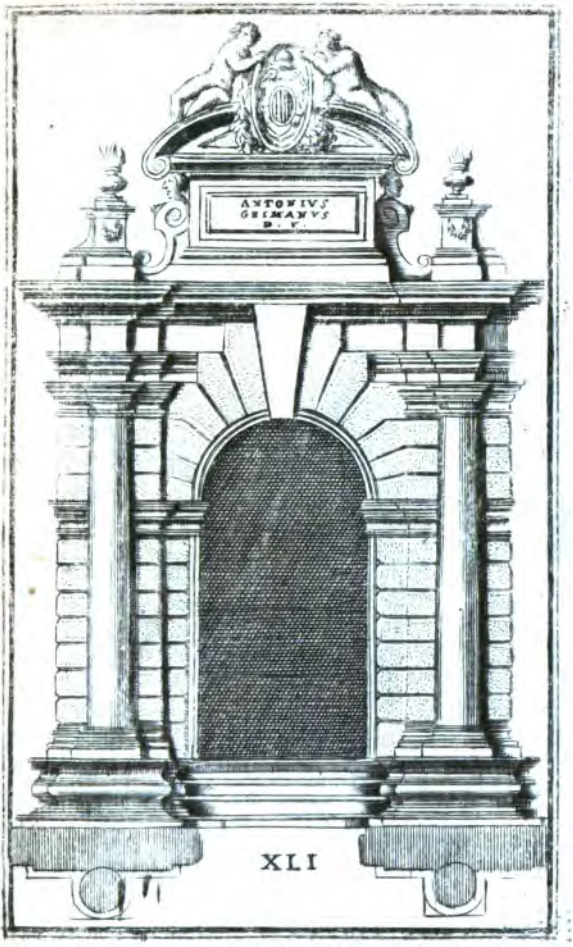
XXXIX

Porte du Iardin du tresſ-illuſtre et tres excellent Seigneur le Duc Sforce

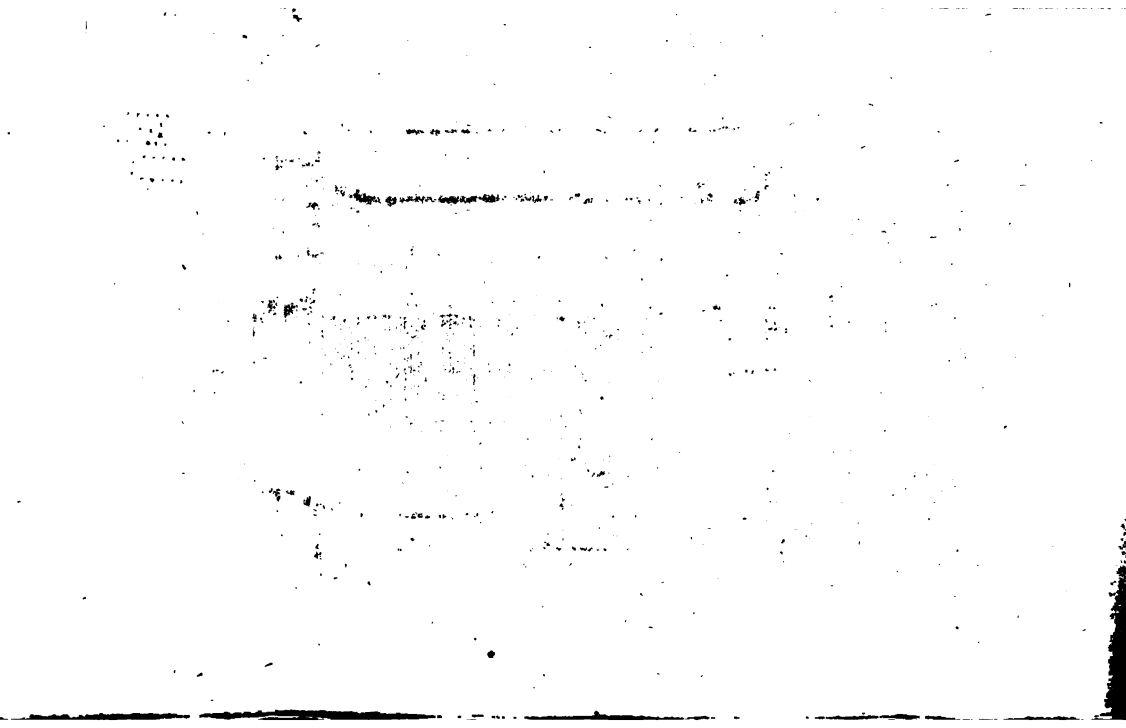


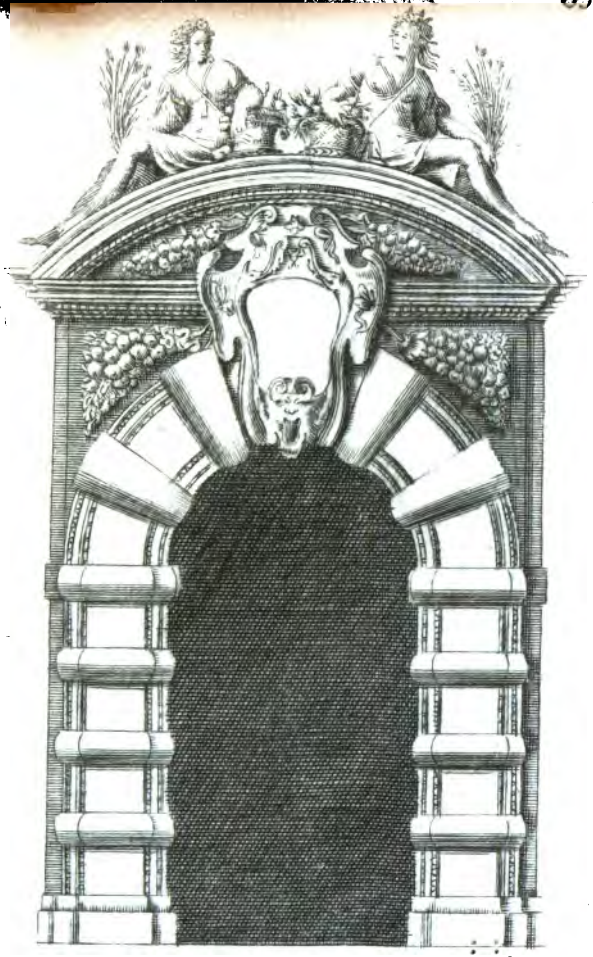
XI

Porte de la Vigne du Reuerendissime Patri=
arche Grimani dans la Voye Pie.



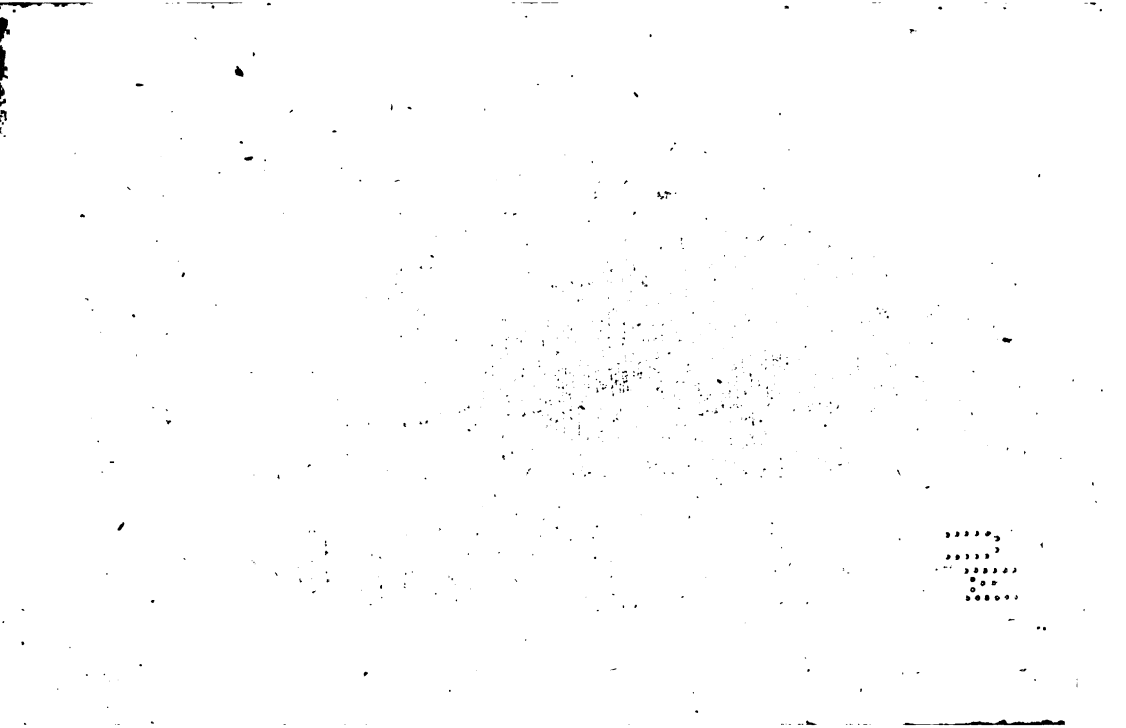
XLI

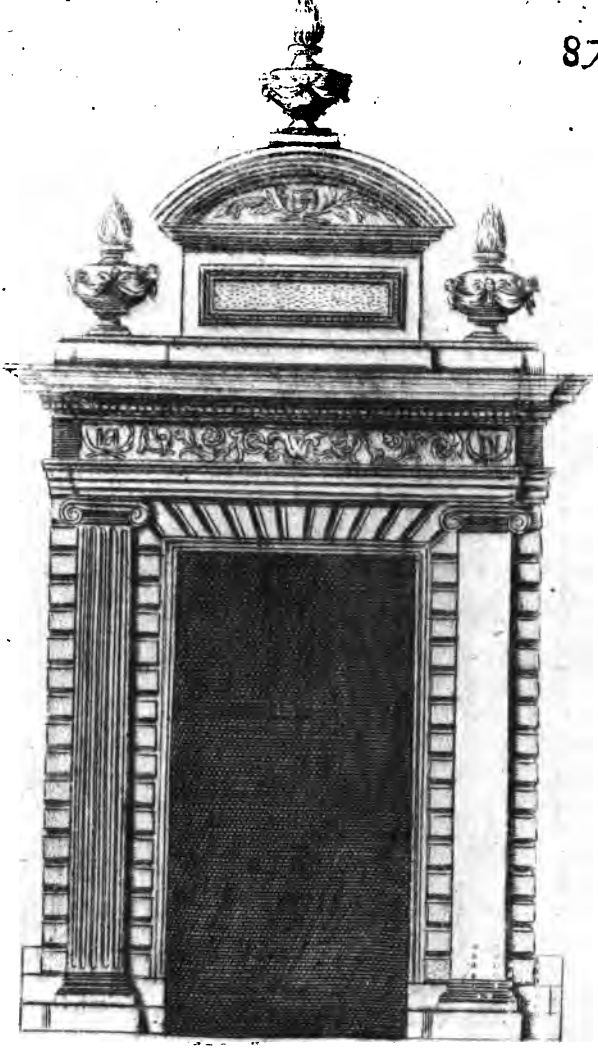




XLII

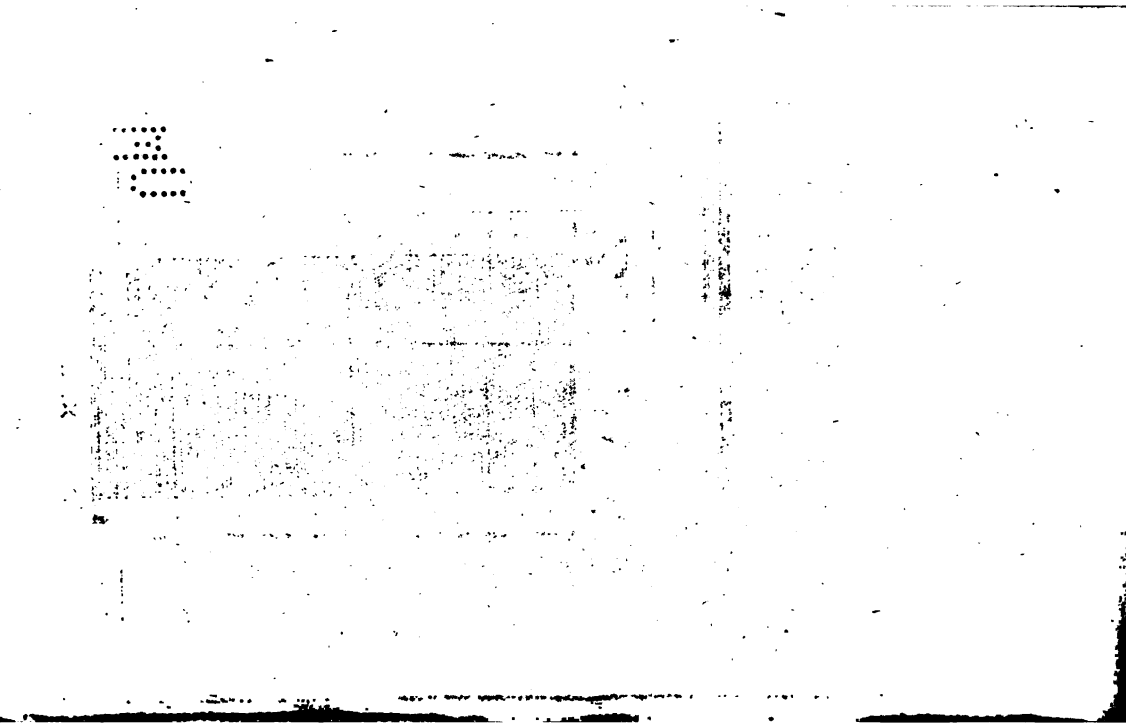
1 2 3 4 5 6 12. P.

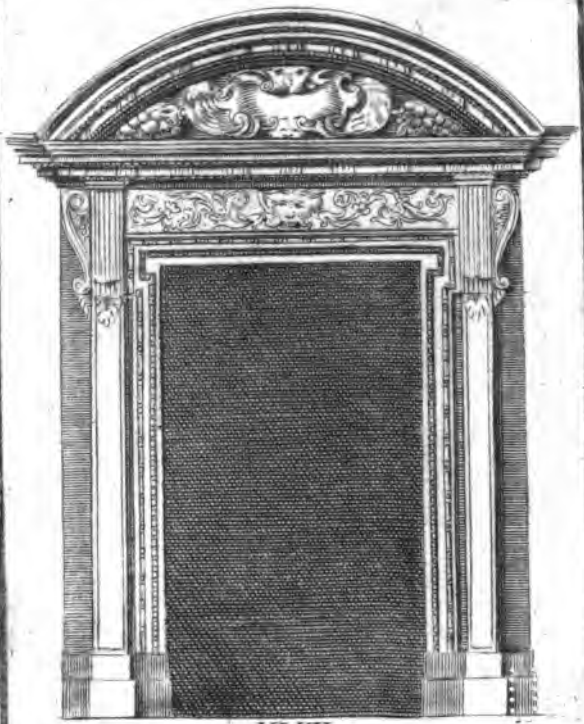


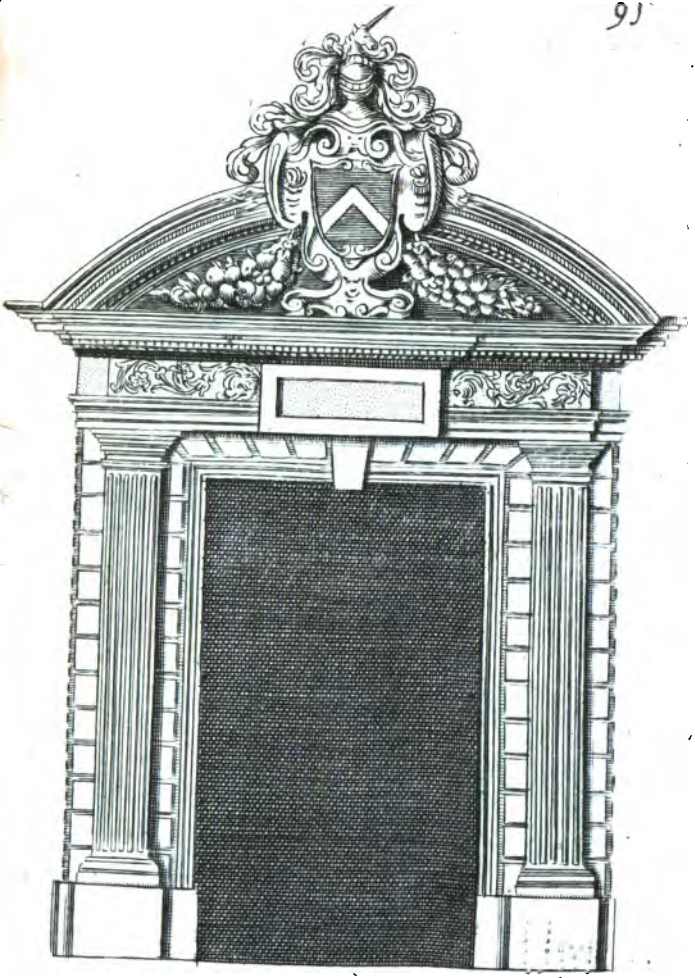


XLIII

1 2 3 4 5 6 12 Pieds

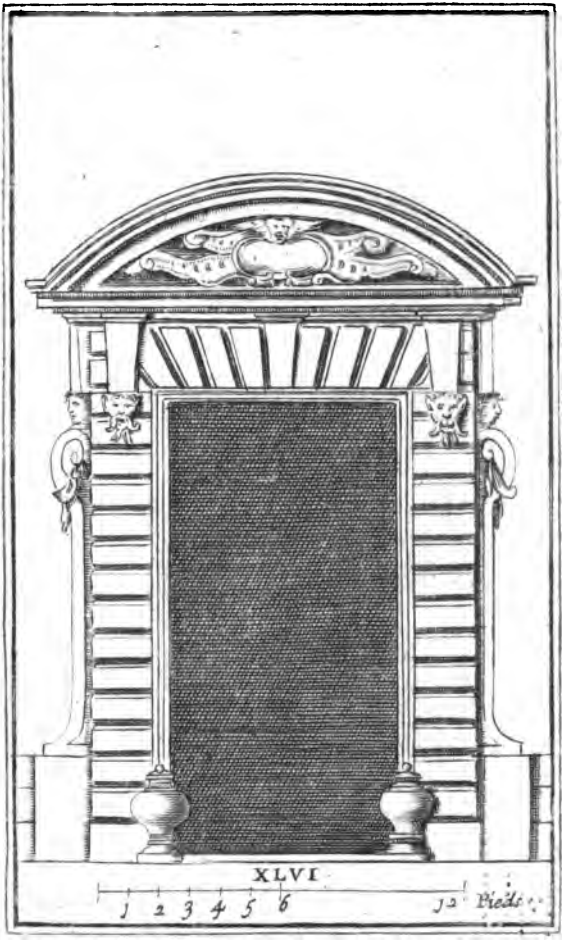


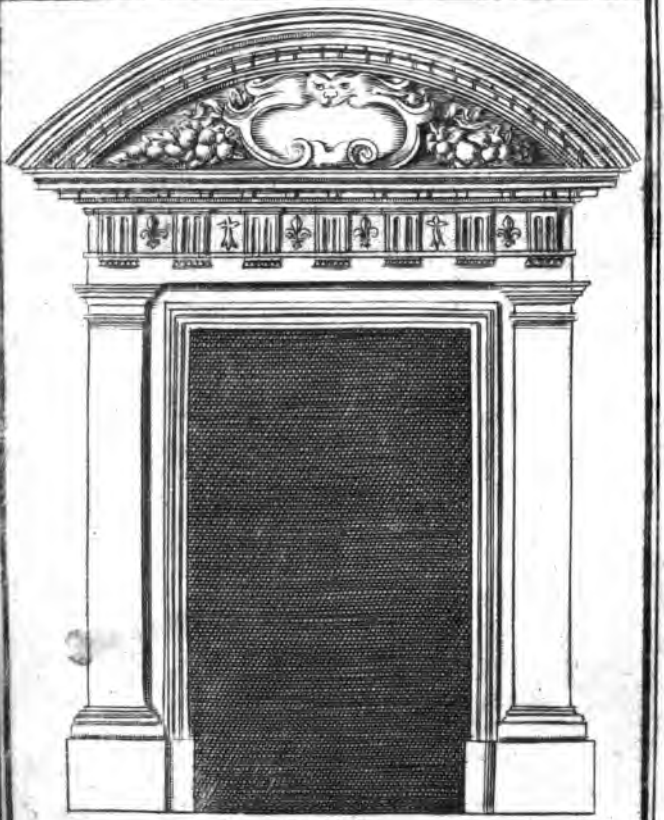

XLIIII
1 2 3 4 5 6
12 P.



XLV.

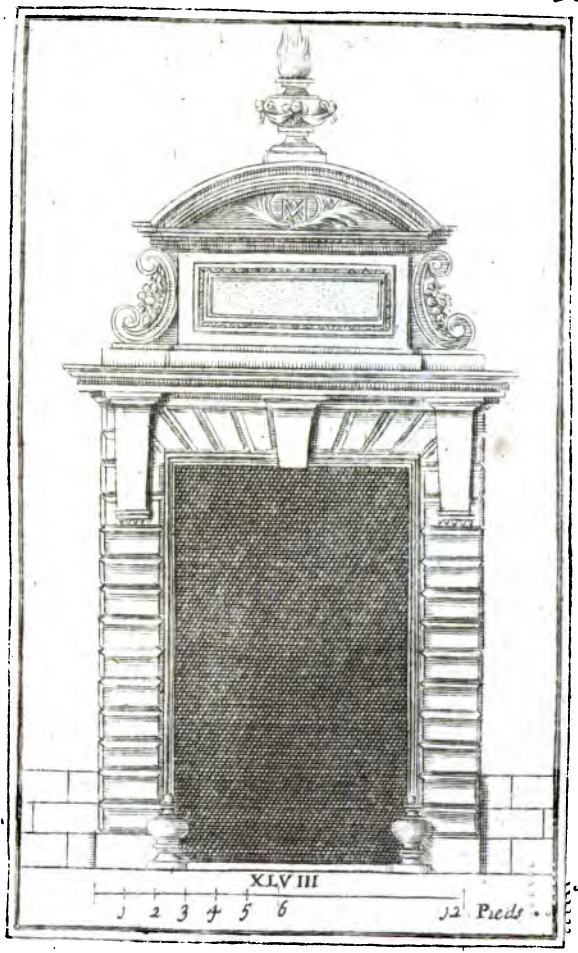
1 2 3 4 5 6 12. Pieds.

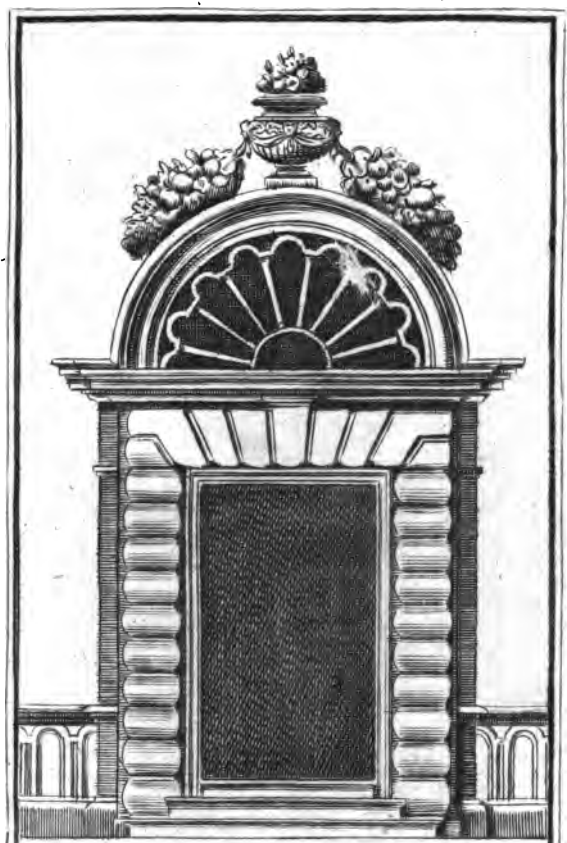
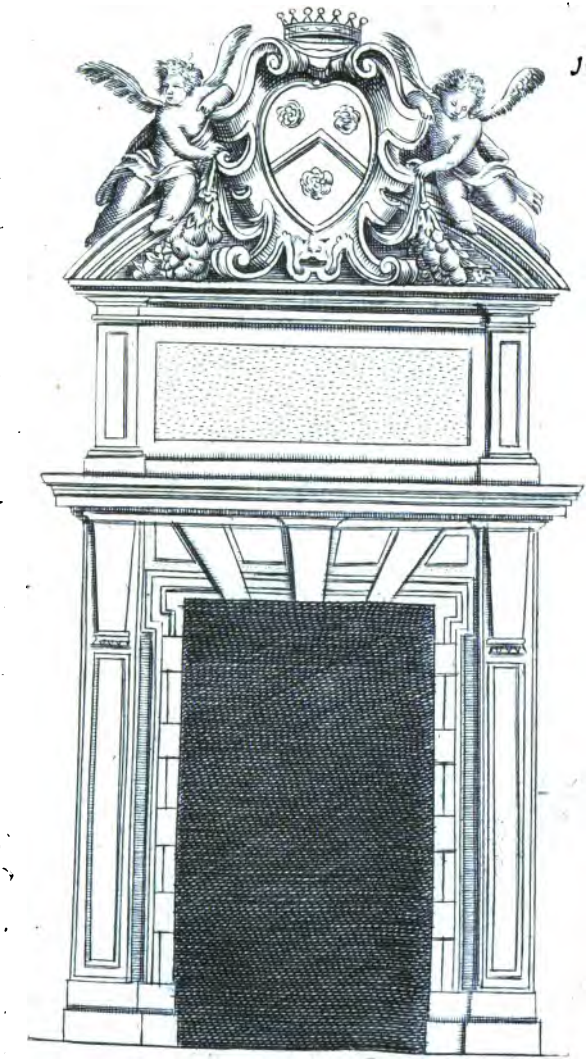

XLVI
1 2 3 4 5 6 12 Pieds



XLVII.

1 2 3 4 5 6 12 Pieds.


XLV III
1 2 3 4 5 6
12 Pieds


XLIX
1 2 3 4 5 6 12 P